Bottom of Form

# The Remainder Theorem and the Factor Theorem

This section of the CAPE MATHEMATICS UNIT 1 syllabus discusses the historical method of solving higher degree polynomial equations.

As we defined in class a **polynomial function** is of the form:

*f*(*x*) = *a*0*xn* + *a*1*xn*-1 + *a*2*xn*-2 + ... + *an*

where

*a*0 ≠ 0 and

n is a positive integer, called the **degree** of the polynomial.

### Example 1

*f*(*x*) = 7*x*5 + 4*x*3 − 2*x*2 − 8*x* + 1 is a polynomial function of degree 5.

## Dividing Polynomials

First, let's consider what happens when we divide numbers.

Say we try to divide `13` by `5`. We will get the answer `2` and have a remainder of `3`. We could write this as:

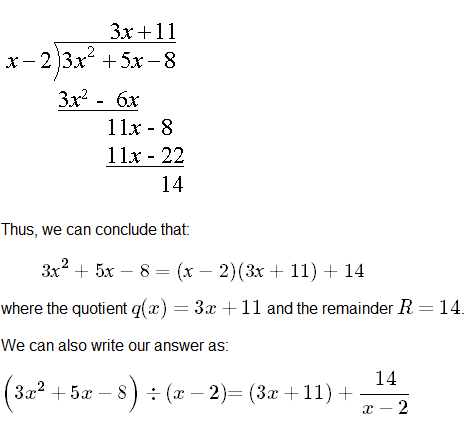
`13/5 = 2 + 3/5`

Another way of thinking about this example is:

`13 = (2 × 5) + 3`

**Division of polynomials** is something like our number example.

If we divide a polynomial by (*x* − *r*), we obtain a result of the form:

*f*(*x*) = (*x* − *r*) *q*(*x*) + *R*

where *q*(*x*) is the quotient and *R* is the remainder.

### Example 2

Divide *f*(*x*) = 3*x*2 + 5*x* − 8 by (*x* − 2).

## The Remainder Theorem

Consider *f*(*x*) = (*x* − *r*)*q*(*x*) + *R*

Note that if we let *x = r*, the expression becomes

*f*(*r*) = (*r* − *r*) *q*(*r*) + *R*

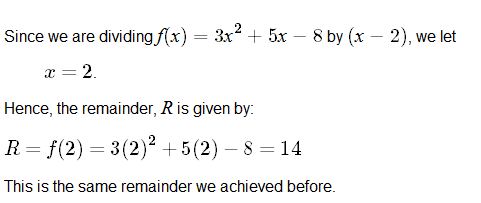
Simplifying gives:

*f*(*r*) = *R*

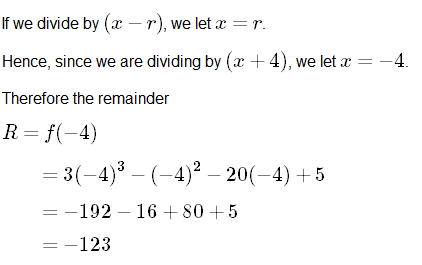
This leads us to the **Remainder Theorem** which states:

If a polynomial *f*(*x*) is divided by (*x* − *r*) and a remainder *R* is obtained, then *f*(*r*) = *R*.

### Example 3

Use the remainder theorem to find the remainder for Example 1 above, which was divide *f*(*x*) = 3*x*2 + 5*x* − 8 by (*x* − 2).

### Example 4

By using the remainder theorem, determine the remainder when

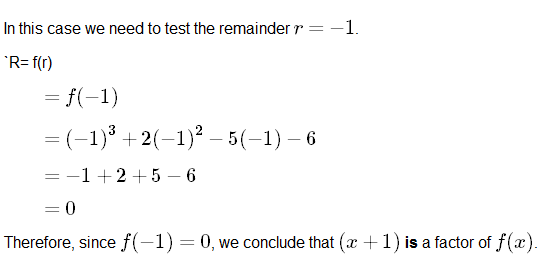
3*x*3 − *x*2 − 20*x* + 5

is divided by (*x* + 4).

## The Factor Theorem

The Factor Theorem states:

If the remainder *f*(*r*) = *R* = 0, then (*x* − *r*) is a factor of *f*(*x*).

The Factor Theorem is powerful because it can be used to find roots of polynomial equations.

### Example 5

Is (*x* + 1) a factor of *f*(*x*) = *x*3 + 2*x*2 − 5*x* − 6?

### Exercises

**1.** Find the remainder *R* by long division **and** by the Remainder Theorem.

(2*x*4 - 10*x*2 + 30*x* - 60) ÷ (*x* + 4) Ans: 172

**2.** Find the remainder using the Remainder Theorem

(*x*4 − 5*x*3 + *x*2 − 2*x* + 6) ÷ (*x* + 4) Ans: 606

**3.** Use the Factor Theorem to decide if (*x* − 2) is a factor of

*f*(*x*) = *x*5 − 2*x*4 + 3*x*3 − 6*x*2 − 4*x* + 8. Ans: YES

**4.** Determine whether `-3/2` is a zero (root) of the function:

*f*(*x*) = 2*x*3 + 3*x*2 − 8*x* − 12. Ans: YES