CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

21 MAY 2018 (a.m.)

M1802134020

FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

TEST CODE | 0 | 2 | 1 | 3 | 4 | 0 | 2 | 0 |

SUBJECT PURE MATHEMATICS – UNIT 1 – Paper 02

PROFICIENCY ADVANCED

REGISTRATION NUMBER | | | | | | | | | | |

SCHOOL/CENTRE NUMBER					

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH | | | | | | | | |

SIGNATURE _____

FORM TP 2018298

TEST CODE 02134020

MAY/JUNE 2018

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 1 – Paper 02

ALGEBRA, GEOMETRY AND CALCULUS

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Each section consists of TWO questions.
3. Answer ALL questions from the THREE sections.
4. Write your answers in the spaces provided in this booklet.
5. Do NOT write in the margins.
6. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.
7. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
8. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

Copyright © 2017 Caribbean Examinations Council
All rights reserved.

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Let **p** and **q** be any two propositions. Complete the truth table below.

p	q	~ p	~ q	(p ∨ q)	~ (p ∨ q)	~ p ∧ ~ q
T	T					
T	F					
F	T					
F	F					

[4 marks]

(ii) Hence, state whether the statements ~ (**p** ∨ **q**) and ~ **p** ∧ ~ **q** are logically equivalent. **Justify your response.**

[1 mark]

(b) A binary operation $\oplus$ is defined on the set of real numbers, R, as $a \oplus b = 2a + 3b$ for all a, b in R.

(i) Calculate $5 \oplus 2$.

[1 mark]

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(ii) Prove that $\oplus$ is closed in R.

[3 marks]

(iii) Determine whether $\oplus$ is commutative.

[3 marks]

(c) Given that $(2x + a)(x - 1)(bx + 1) \equiv cx^3 + 10x^2 - 2x - 10$ is a polynomial identity in x, calculate the values of a, b and c.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

[7 marks]

(d) Solve the logarithmic equation $log_4 (2x + 2) - log_2 (x + 1) = 1$.

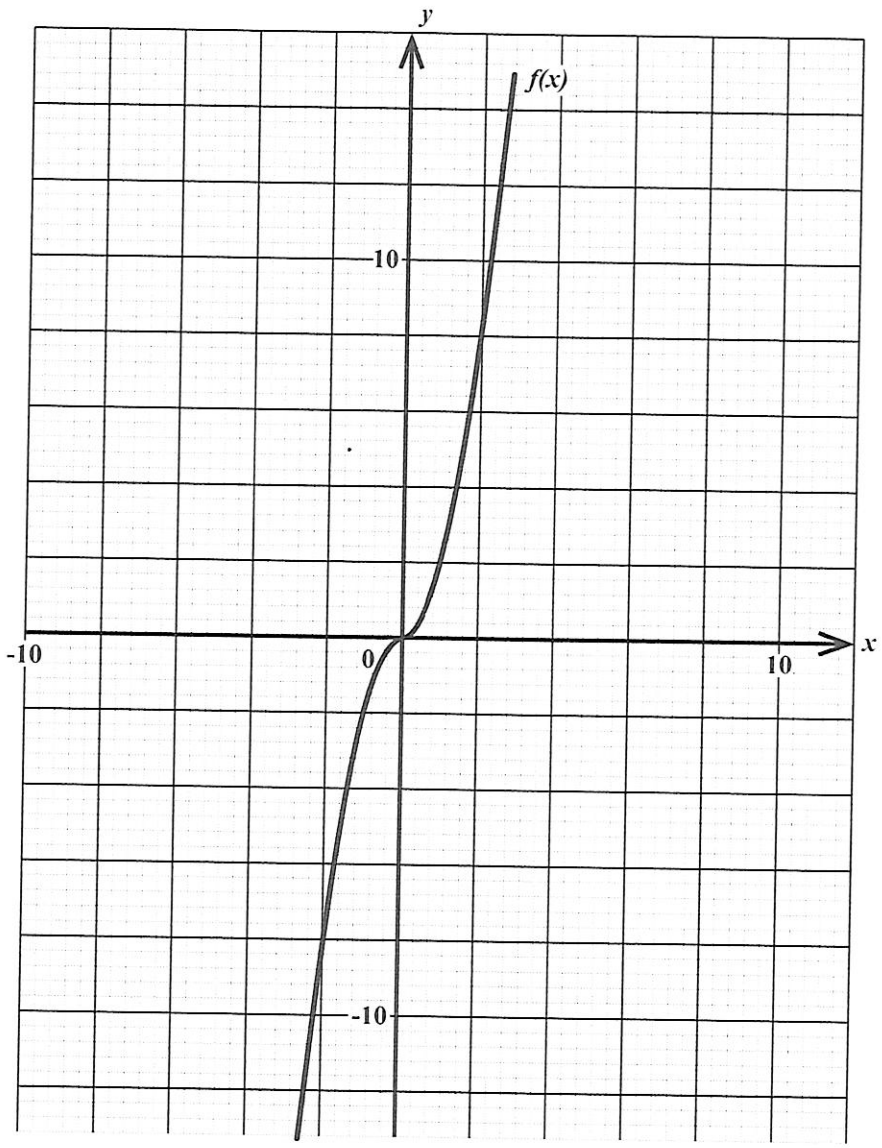
[6 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

2. The diagram below shows the graph of the function $f(x)$.



(a) **On the diagram above,**

(i) sketch the inverse of $f(x)$. **[2 marks]**

(ii) Use a graphical method to show that f is bijective. **[3 marks]**

(b) (i) Prove that $|x-y| \leq |x-z| + |z-y|$ for all $x, y, z \in \mathbf{R}$.

[4 marks]

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(ii) Solve the inequality $|6x - 2| + x^2 \leq 5$.

[8 marks]

(c) Given that α, β and γ are the roots of the equation $2x^3 - x^2 + 1 = 0$, determine the equation whose roots are $\frac{1}{\alpha^2}$, $\frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$.

Note: $(\alpha\beta)^2 + (\alpha\gamma)^2 + (\beta\gamma)^2 = (\alpha\gamma + \beta\alpha + \gamma\alpha)^2 - 2\,\alpha\beta\gamma(\alpha + \beta + \gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

[8 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Show that $\dfrac{\sin 2\theta - \cos 2\theta + 1}{\cos 2\theta + \sin 2\theta - 1} = \sec 2\theta + \tan 2\theta$.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

[8 marks]

(ii) Hence, or otherwise, determine the general solution of $\frac{\sin\theta - \cos\theta + 1}{\cos\theta + \sin\theta - 1} = 0$.

[5 marks]

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(b) Given that $\cos A = \frac{3}{5}$ and $\sin B = \frac{3}{4}$, where A and B are acute angles, calculate

(i) $\sin 2A$

[3 marks]

(ii) $\cos (A + B)$.

[3 marks]

(c) Solve the equation $\sin\theta - \sqrt{3}\cos\theta = 1$, for $-\pi \leq \theta \leq \pi$.

[6 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) The equations of the circle, C, and a straight line, L, are given by

$x^2 + y^2 - 2x + 2y + 1 = 0$ and $y = x - 3$, respectively.

(i) Determine the centre and radius of the circle, C.

[3 marks]

(ii) Show that $(1, -2)$ is one of the points of intersection of the circle, C, and the straight line, L.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

[6 marks]

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(iii) Determine the equation of the tangent to the circle, C, at the point $(1, -2)$.

[4 marks]

(b) Let $\mathbf{u} = \begin{pmatrix} s \\ 3 \\ s \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix}$ be two position vectors in R^3.

Show that $\mathbf{u}$ and $\mathbf{v}$ are NOT parallel.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

[9 marks]

GO ON TO THE NEXT PAGE

(c) Determine the vector equation of a plane which passes through the point (1, 3, 0) and which is perpendicular to the vector $2i + 4j + 5k$.

[3 marks]

Total 25 marks

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

SECTION C

Module 3

Answer BOTH questions.

5. (a) Use the substitution $u = x^4 + 2$ to determine $\int (x^4 + 2)^3 (4x^7)\, dx$.

[6 marks]

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(b) Calculate the area of the region lying between the parabolas $y = x^2$ and $x = \frac{1}{8} y^2$.

[6 marks]

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(c) The equation of a curve is given as $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x$.

(i) Determine f'.

[1 mark]

(ii) Determine f''.

[1 mark]

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(iii) Calculate the x coordinates of the stationary points of $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x$ and determine the nature of these stationary points.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

[11 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) A function f is defined as

$$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1} & x < 1 \\ 4x & x > 1 \\ 2 & x = 1. \end{cases}$$

(i) Determine whether or not the limit of f at $x = 1$ exists.

[4 marks]

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(ii) Determine whether f is continuous at $x = 1$.

[2 marks]

(b) The parametric equation of a curve, g, is given by $x = 2 \cos \theta$ and $y = 3 - \sin \theta$.

(i) Determine $\frac{d}{dx} g(x)$, in terms of θ.

[3 marks]

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(ii) Determine the equation of the normal to the curve at the point $\left(\sqrt{3}, \frac{5}{2}\right)$.

[8 marks]

GO ON TO THE NEXT PAGE

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

(c) The slope of a tangent of the curve, g, at any point (x, y) is equal to the product of the x-coordinate and the reciprocal of the y-coordinate of the point of tangency.

(i) Formulate an appropriate differential equation and find the equation of the curve family.

[5 marks]

(ii) Hence, or otherwise, determine the equation of the curve given that it passes through the point (1, 3).

[3 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA