FORM TP 2004248

TEST CODE **02234010**

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 2 – PAPER 01

$1\frac{1}{2}$ hours

24 MAY 2004 (p.m.)

This examination paper consists of THREE sections: Module 2.1, Module 2.2 and Module 2.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

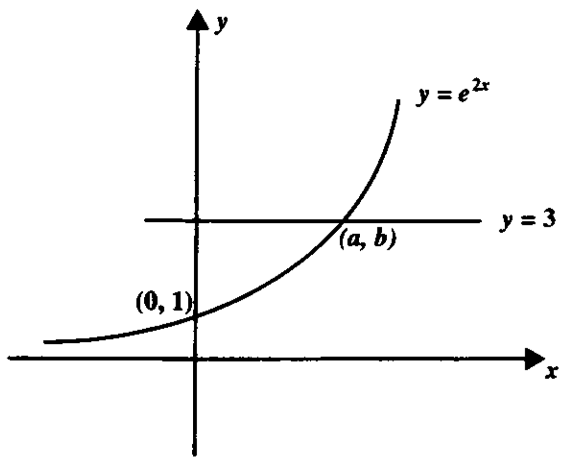
Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 2.1)

Answer ALL questions.

1. In the diagram below (**not drawn to scale**), the line $y = 3$ cuts the curve $y = e^{2x}$ at the point (a, b).



Calculate the values of a and b. **[4 marks]**

2. Differentiate, with respect to x

(a) $y = ln(3x^2),\ x \neq 0.$ **[2 marks]**

(b) $y = \sin^2 x \cos x.$ **[3 marks]**

3. (a) Find the gradient at the point (1,1) on the curve $2xy + y^2 - 3 = 0.$ **[5 marks]**

(b) Solve, for x, the equation $e^{2x} - 3e^x - 4 = 0.$ **[3 marks]**

4. (a) Express in partial fractions

$$\frac{x+1}{x(x+2)}.$$ **[5 marks]**

(b) Hence, find $\int \frac{x+1}{x(x+2)}\, dx\ ,\ x > 0.$ **[3 marks]**

5. Find $\int x^2 \ln x\ dx\ ,\ x > 0.$ **[5 marks]**

Total 30 marks

GO ON TO THE NEXT PAGE

SECTION B (MODULE 2.2)

Answer ALL questions.

6. Find the term independent of x in the expansion of $(3x - \frac{1}{2x^2})^9$. **[5 marks]**

7. The first four terms of an A.P. are 2, 5, $2x + y + 7$ and $2x - 3y$ respectively, where x and y are constants. Find the values of x and y. **[8 marks]**

8. (a) Find the sum to n terms of the geometric series

$$4 + 2 + 1 + \frac{1}{2} + \ldots$$ **[5 marks]**

(b) Deduce the sum to infinity of the series. **[2 marks]**

9. If $(1 + ax)^n \equiv 1 + 6x + 16x^2 + \ldots$ find the values of the constants a and n. **[6 marks]**

10. A craftsman estimated the side of a square tile to be 16 cm, but the actual measurement was 16.14 cm. Calculate, to two decimal places, the percentage error in the actual measurement of the area of the tile. **[4 marks]**

Total 30 marks

SECTION C (MODULE 2.3)

Answer ALL questions.

11. A bag contains 2 red balls, R_1 and R_2, 1 green ball, G, and 2 black balls, B_1 and B_2. Randomly, two balls are drawn together from the bag.

(a) Describe the sample space. **[2 marks]**

(b) Determine the probability that

(i) BOTH balls are the same colour **[2 marks]**

(ii) AT LEAST ONE ball is black. **[3 marks]**

12. Only three horses, A, B and C are in a race. The probability that A wins the race is twice the probability that B wins. The probability that B wins the race is twice the probability that C wins. Find the probability of winning for EACH of the horses.
[5 marks]

13. Let A and B be the events such that $P(A \cup B) = \frac{3}{4}$, $P(A') = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$.

Find:

(a) $P(A)$ **[1 mark]**

(b) $P(B)$ **[3 marks]**

(c) $P(A \cap B')$ **[3 marks]**

14. (a) Which ONE of the following situations describes a mutually exclusive event?

(i) Selecting either an even or a prime number from the set of real numbers.

(ii) Selecting either a negative integer or perfect square from the set of integers .

(iii) Selecting either a perfect square or an odd number from the numbers 1 to 100.
[1 mark]

(b) The probability that A hits a target is $\frac{1}{4}$ and the probability that B hits the same target is $\frac{2}{5}$. The event that A hits the target is independent of the event that B hits the target. What is the probability that both A and B hit the target? **[5 marks]**

15. A marksman shoots at a target and he either hits the target (H) or misses it (M). The probability of H is 0.4. He shoots at the target 3 times.
Determine

(a) the elements of the event A associated with the marksman hitting the target EXACTLY twice **[2 marks]**

(b) the probability that the marksman hits the target AT LEAST once. **[3 marks]**

Total 30 marks

END OF TEST

FORM TP 2004249

TEST CODE **02234020**

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 2 – PAPER 02

$2\frac{1}{2}$ hours

02 JUNE 2004 (p.m.)

This examination paper consists of THREE sections: Module 2.1, Module 2.2 and Module 2.3.

Each section consists of 2 questions.
The maximum mark for each section is 50.
The maximum mark for this examination is 150.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 2.1)

Answer BOTH questions.

1. (a) If $2\log_a 2 + \log_a 10 - 3\log_a 3 = 3 + \log_a 5$, $a > 0$, find the value of a. **[5 marks]**

(b) Find the value(s) of $x \in \boldsymbol{R}$ which satisfy $2\log_3 x = \log_3(x+6)$. **[5 marks]**

(c) Complete the table below for values of 2^x and e^x using a calculator, where necessary. Approximate all values to 1 decimal place.

x	-2	-1	0	0.5	1	1.5	2	2.5	3
2^x	0.3			1.4		2.8		5.7	
e^x	0.1			1.6		4.5		12.2	

[4 marks]

(d) On the same pair of axes and using a scale of 2 cm for 1 unit on the x-axis, 1 cm for 1 unit on the y-axis, draw the graphs of the two curves
$y = 2^x$ and $y = e^x$ for $-2 \leq x \leq 3$. **[7 marks]**

(e) Use your graphs to find

(i) the value of x satisfying $2^x = e^x$ **[2 marks]**

(ii) the SMALLEST INTEGER x for which $e^x - 2^x > 3$. **[2 marks]**

Total 25 marks

2. (a) Differentiate, with respect to x, EACH function below. Simplify your answers as far as possible.

(i) $\dfrac{e^x}{x+1}$ **[5 marks]**

(ii) $\tan^2(x^3)$. **[5 marks]**

(b) Use the substitution $u = \sin x$ to find

$\int e^{\sin x} \cos x \, dx$. **[5 marks]**

(c) The parametric equations of a curve are given by
$x = 3 - 2t$, $y = t(1-t)$.

(i) Find $\dfrac{dy}{dx}$ in terms of t. **[4 marks]**

(ii) A normal to the curve is parallel to the line $x + y = 2$. Find the equation of this normal. **[6 marks]**

Total 25 marks

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SECTION B (MODULE 2.2)

Answer BOTH questions.

3. (a) A sequence of real numbers $\{u_n\}$ satisfies the recurrence relation:

$$u_1 = 1 , \ u_n u_{n+1} = 2 .$$

(i) Show that $u_{n+2} = u_n$. **[2 marks]**

(ii) Given that $a_n = u_{n+1} + u_n$ and $b_n = u_{n+1} - u_n$, write down the first FOUR terms of each of the sequences $\{u_n\}$, $\{a_n\}$ and $\{b_n\}$. **[6 marks]**

(iii) State which of the sequences in (ii) above is convergent, divergent or periodic. **[3 marks]**

(b) Prove by mathematical induction that

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3} n(n+1)(n+2) \text{, for all } n \in \mathbf{N}.$$ **[9 marks]**

(c) Find the sum of the arithmetic progression
72, 69, 66, ..., -24, -27. **[5 marks]**

Total 25 marks

4. (a) (i) The function $f\colon \boldsymbol{R} \to \boldsymbol{R}$ is given by $f(x) = x^3 + 2x - 2$.
Show that

a) f is a strictly increasing function **[3 marks]**

b) the equation $f(x) = 0$ has a root α in the interval [0, 1] **[3 marks]**

c) the equation $f(x) = 0$ has **no other root** in the interval [0, 1]. **[3 marks]**

(ii) By starting with $x_1 = 0.5$ as a first approximation to the root, α, use the Newton-Raphson method to find a second approximation, x_2, to the root α correct to 3 decimal places. **[4 marks]**

(b) Given that the coefficient of x^2 is zero in the binomial expansion of $(1 - ax)(1 + 2x)^5$, find the value of a and the coefficient of x^3. **[12 marks]**

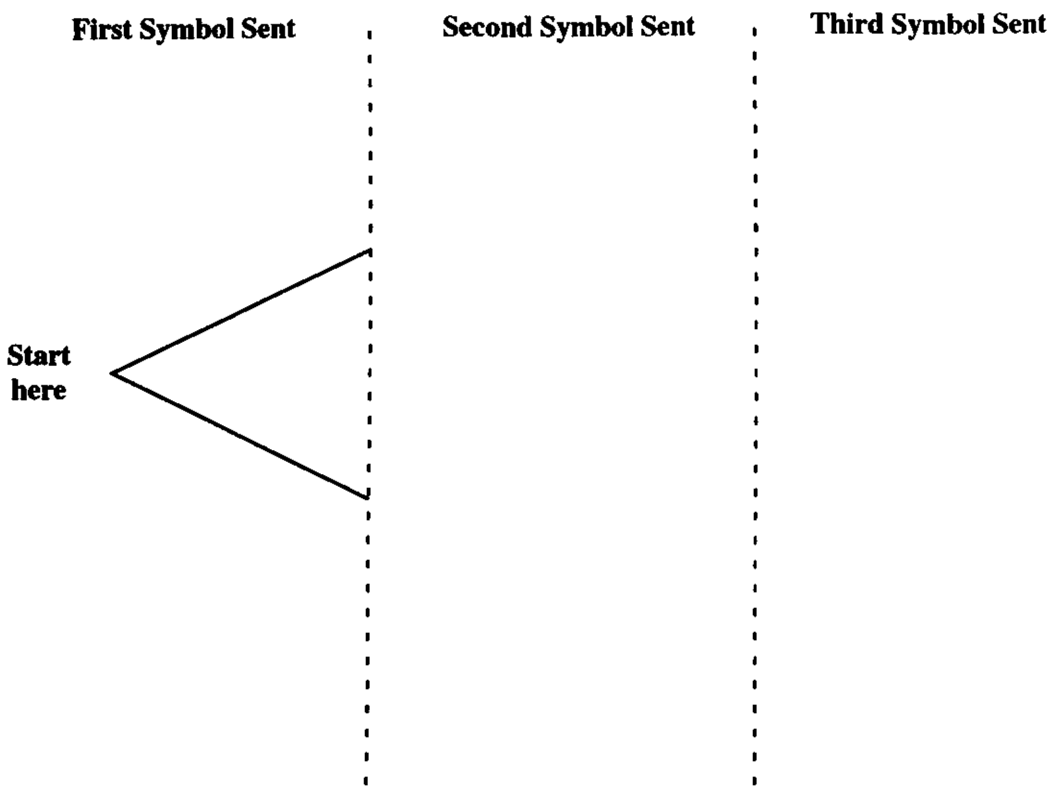
Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C (MODULE 2.3)

Answer BOTH questions.

5. (a) A message is sent using two symbols, α and β, arranged in sequence. The probability that the first symbol sent is α, is $\frac{1}{5}$.
A tree diagram is started in the diagram below.



Copy the diagram in your answer booklet and use the information above to write the correct probability on EACH branch of the tree diagram. **[2 marks]**

(b) After the first symbol has been sent, the probability that an α is sent is $\frac{1}{4}$ if the preceding symbol was an α and $\frac{1}{3}$ if the preceding symbol was a β.

(i) Use this information to extend your diagram to represent the FIRST THREE symbols sent. **[3 marks]**

(ii) Write CLEARLY the probabilities on EACH branch of your tree diagram. **[8 marks]**

(c) Using the information from your tree diagram, find the probability that

(i) there will be EXACTLY TWO α's among the FIRST THREE symbols sent **[6 marks]**

(ii) the THREE symbols sent are identical. **[6 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) A rectangular container with a lid, is made from thin metal. Its length is $2x$ metres and its width is x metres. The box must have a volume of 72 cubic metres.

(i) Show that the area, A square metres, of metal used is given by $A = 4x^2 + \frac{216}{x}$. **[5 marks]**

(ii) Find the value of x so that A is a minimum. **[5 marks]**

(b) The cost of making x articles per day is $\$(\frac{1}{2}x^2 + 50x + 50)$ and the selling price of each article is $\$(80 - \frac{1}{4}x)$.
Find

(i) the daily profit in terms of x **[5 marks]**

(ii) the value of x to give a maximum profit **[3 marks]**

(iii) the maximum value of the profit. **[2 marks]**

(c) A plot of land is rented on the understanding that the rent for the first year will be \$64 and in subsequent years will always be $\frac{7}{8}$ of what it was the year before. Calculate, **to the nearest dollar**, the TOTAL amount of rent paid for the first 15 years. **[5 marks]**

Total 25 marks

END OF TEST

FORM TP 2004272

TEST CODE **02234032**

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 2 - PAPER 03/2

$1\frac{1}{2}$ hours

24 MAY 2004 (p.m.)

This examination paper consists of THREE questions, one question from each of Modules 2.1, 2.2 and 2.3.

The maximum mark for each question is 20.
The maximum mark for this examination is 60.
This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL THREE** questions.
3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and a pair of compasses

SECTION A (MODULE 2.1)

Answer this question.

1. (a) (i) Complete the table shown below for $y = \frac{1}{2}(e^x + e^{-x})$ using a calculator where necessary. **[5 marks]**

$$y = \frac{1}{2}(e^x + e^{-x})$$

x	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0
e^x	0.14	0.22		0.61		1.65		4.48	7.40
e^{-x}	7.40	4.48		1.65		0.61		0.22	0.14
y	3.27	2.35		1.13		1.13		2.35	3.27

(ii) Hence, sketch the curve $y = \frac{1}{2}(e^x + e^{-x})$ for values of x from -2 to +2. **[4 marks]**

(b) Find $\frac{dy}{dx}$. **[2 marks]**

(c) Show that $1 + \left(\frac{dy}{dx}\right)^2 = y^2$. **[4 marks]**

(d) Show that $\int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\ dx = e^2 - \frac{1}{e^2}$. **[5 marks]**

Total 20 marks

SECTION B (MODULE 2.2)

Answer this question.

2. In a model for the growth of a population, p_n is the number of individuals in the population at the end of n years. Initially, the population consists of 1000 individuals.

   In each calendar year, the population increases by 20%, and on December 31st, 100 individuals leave the population.

   (a) Calculate the values of p_1 and p_2. **[4 marks]**

   (b) Write down an equation connecting p_{n+1} and p_n. **[2 marks]**

   (c) Show by Mathematical Induction, or otherwise, that $p_n = 500\,(1.2)^n + 500$. **[9 marks]**

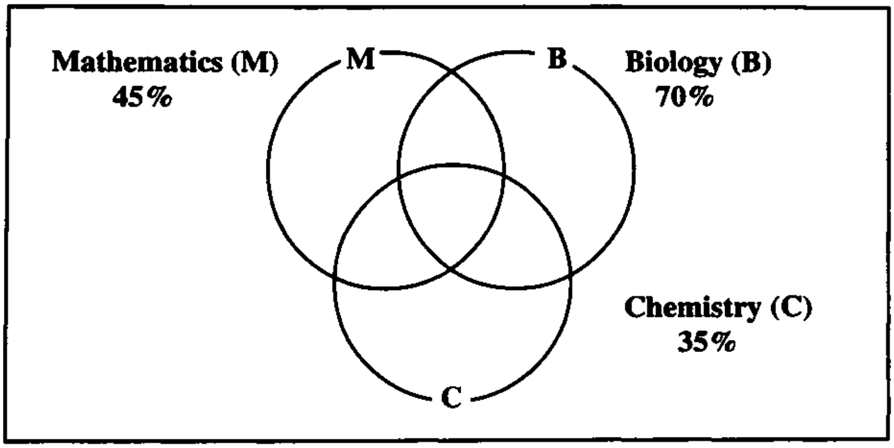
   (d) Calculate the **smallest** value of n for which $p_n > 10\,000$. **[5 marks]**

Total 20 marks

SECTION C (MODULE 2.3)

Answer this question.

3. (a) In a sixth form, the students are studying one or more of the subjects, Biology (B), Chemistry (C) and Mathematics (M). 15% of the students are studying both Chemistry and Mathematics and 3% of them are studying all three subjects. Some of this information is shown on the diagram below.



(i) What is the probability that a student chosen at random is studying Chemistry? **[1 mark]**

(ii) Given that a student is studying Chemistry, what is the probability that the student is also studying Mathematics? **[3 marks]**

(iii) Find the probability that a student who is studying Chemistry and Mathematics is also studying Biology. **[3 marks]**

(b) A and B are two independent events such that $P(A) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find

(i) $P(B)$ **[4 marks]**

(ii) $P(A \cup B)$. **[4 marks]**

(c) A three-digit number is formed by choosing, with replacement, three digits at random from the digits 1, 2, 3, 4, 5. What is the probability that the number formed is divisible by 5? **[5 marks]**

Total 20 marks

END OF TEST

FORM TP 2005255

TEST CODE **02234010**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 01

2 hours

23 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. (a) The function $f(x)$ is defined by $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ where $a_1, a_2, a_3 \in \mathbf{R}$.

   (i) Obtain $f'(x)$. **[1 mark]**

   (ii) Given that $f'(x) = \lambda f(x), \lambda \neq 0, \lambda \in \mathbf{R}$, express the coefficients a_1, a_2, a_3, in terms of λ and a_0. **[4 marks]**

   (b) Differentiate with respect to x the following:

   (i) $y = e^{\cos x}$ **[2 marks]**

   (ii) $y = x^2\sqrt{1+x}$ **[3 marks]**

Total 10 marks

2. (a) Given $\log_{10}3 = m$, $\log_{10}e = n$, express in terms of m and/or n

   (i) $\log_{10}\left(\frac{3}{10}\right)$ **[3 marks]**

   (ii) $\log_e 9$. **[4 marks]**

   (b) Find the value of $x \in \mathbf{R}$ for which $3^x = 7$. **[3 marks]**

Total 10 marks

3. Given that $u = \tan\theta$,

(a) use the quotient rule to show that $\frac{du}{d\theta} = \sec^2\theta$ **[4 marks]**

(b) find $\int \tan^3\theta \sec^2\theta \, d\theta$. **[3 marks]**

Total 7 marks

4. Show that, if $y = \sin x \cos x$ then

(a) $\frac{dy}{dx} = 2\cos^2 x - 1$ **[3 marks]**

(b) $\frac{d^2y}{dx^2} + 4y = 0.$ **[3 marks]**

Total 6 marks

5. (a) Find $\int \frac{6x}{x^2 + 1} \, dx.$ **[3 marks]**

(b) By using the substitution $u = x^2$, or otherwise, find $\int x e^{-x^2} \, dx$. **[4 marks]**

Total 7 marks

Section B (Module 2)

Answer ALL questions.

6. A sequence $\{u_n\}$ is generated by the recurrence relation

$$u_{n+1} = \frac{8}{u_n - 3},\ n \geq 1.$$

(a) If $u_2 = 2u_1$, find the possible values of u_1. **[4 marks]**

(b) Find u_3 corresponding to EACH value of u_1. **[4 marks]**

Total 8 marks

7. The sum, S_n, of the first n terms of a series is given by $S_n = 2n\,(n - 2)$.

(a) Find, in terms of n, the nth term of the series. **[4 marks]**

(b) Hence, show that the series is an AP. **[2 marks]**

(c) For the AP in (b) above, identify

(i) the first term **[1 mark]**

(ii) the common difference. **[1 mark]**

Total 8 marks

8. (a) Express the binomial coefficients ${}^nC_k$ and ${}^nC_{k-1}$, for $n > k \geq 1$, in terms of factorials. **[2 marks]**

(b) Hence, show that

(i) ${}^nC_k = {}^nC_{n-k}$ **[2 marks]**

(ii) ${}^nC_k + {}^nC_{k-1} = {}^{n+1}C_k$. **[6 marks]**

Total 10 marks

9. The coefficient of x^7 is sixteen times the coefficient of x^{11} in the expansion of $(a + x)^{18}$. Given that ${}^nC_k = {}^nC_{n-k}$, find the possible value(s) of the real constant a. **[7 marks]**

Total 7 marks

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10. The length of one side of a square tile is given as 25 cm correct to the nearest cm.

(a) Within what limits does the length of the tile lie? **[1 mark]**

(b) Calculate

(i) the SMALLEST absolute error in the area of the tile **[3 marks]**

(ii) the MAXIMUM percentage error in the area of the tile. **[3 marks]**

Total 7 marks

Section C (Module 3)

Answer ALL questions.

11. (a) How many four-digit numbers can be formed from the digits 1, 2, 3, 4, 5 if **ALL** digits can be repeated? **[2 marks]**

(b) Determine

(i) how many of the four-digit numbers in (a) above are even **[4 marks]**

(ii) the probability that a four-digit number in (a) above is odd. **[2 marks]**

Total 8 marks

12. A fair coin is tossed three times.

(a) Determine the sample space. **[3 marks]**

(b) Calculate

(i) the probability that two heads and one tail appear **[2 marks]**

(ii) the probability that **AT LEAST** two heads appear. **[2 marks]**

Total 7 marks

13. The matrices A and B are given below.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ 2 & 2 \\ 2 & -3 \end{pmatrix}$$

(a) Calculate

(i) AB **[2 marks]**

(ii) $B^T A^T$. **[3 marks]**

(b) Deduce that $(AB)^T = B^T A^T$. **[2 marks]**

Total 7 marks

14. (a) Write the augmented matrix for the following system of equations:

$$x + 2y + 3z = 7$$
$$2x + 2y - z = 0$$
$$3x - 4y + 2z = 7$$

[2 marks]

(b) Reduce the augmented matrix obtained to echelon form. **[4 marks]**

(c) Hence, solve the system of equations. **[3 marks]**

Total 9 marks

15. The rate of change of the volume, V, of a sphere of radius r with respect to time, t, is directly proportional to the volume of the sphere.

(a) Obtain a differential equation involving the radius, r, of the sphere. **[5 marks]**

(b) Hence, show that $r = C e^{\frac{kt}{3}}$, $C, k \in \mathbf{R}$. **[4 marks]**

$$[\, V = \frac{4}{3}\pi r^3 \,]$$

Total 9 marks

END OF TEST

FORM TP 2005256

TEST CODE **02234020**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

01 JUNE 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

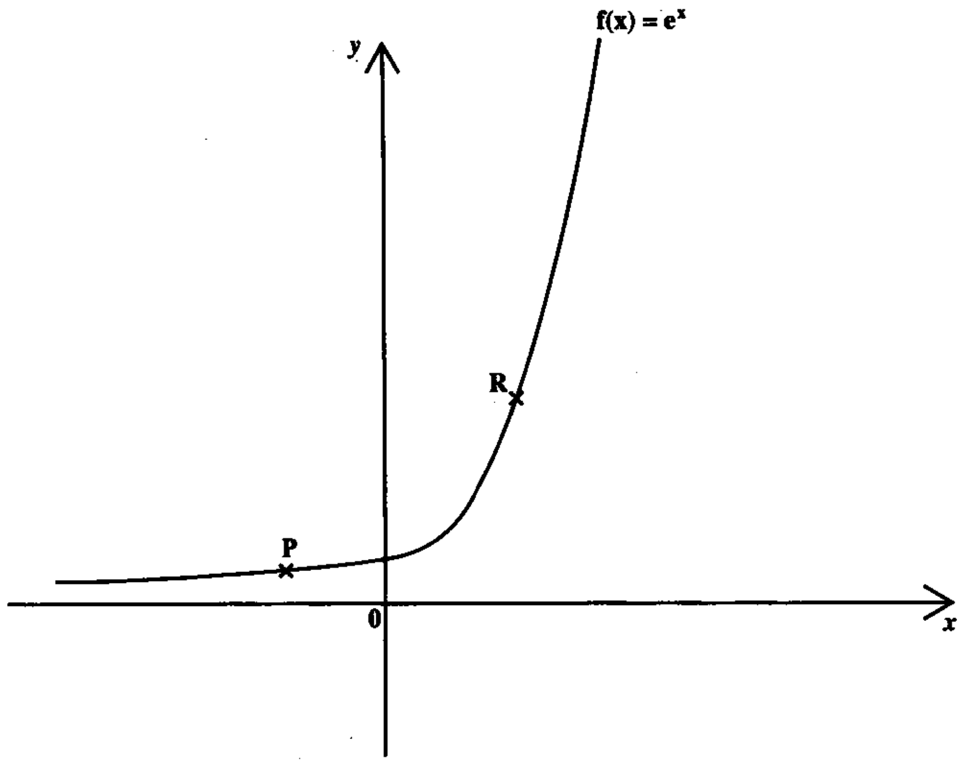
Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) The diagram below, **not drawn to scale**, shows two points, $P(p, 0.368)$ and $R(3.5, r)$, on $f(x) = e^x$ for $x \in \mathbf{R}$.



(i) Copy the diagram above and on the same axes, sketch the graph of $g(x) = \ln x$. **[3 marks]**

(ii) Describe clearly the relationship between $f(x) = e^x$ and $g(x) = \ln x$. **[3 marks]**

(iii) Using a calculator, find the value of

a) r **[1 mark]**

b) p. **[2 marks]**

(b) Given that

$\log_a(bc) = x$, $\log_b(ca) = y$, $\log_c(ab) = z$ and $a \neq b \neq c$,

show that $a^x b^y c^z = (abc)^2$. **[3 marks]**

(c) Find the values of $x \in \mathbf{R}$ for which $e^x + 3e^{-x} = 4$. **[8 marks]**

Total 20 marks

2. (a) A curve is given parametrically by $x = (3 - 2t)^2$, $y = t^3 - 2t$. Find

(i) $\frac{dy}{dx}$ in terms of t **[4 marks]**

(ii) the gradient of the normal to the curve at the point $t = 2$. **[2 marks]**

(b) (i) Express $\frac{2x + 1}{x^2(x + 1)}$ in the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 1}$, where A, B and C are constants. **[7 marks]**

(ii) Hence, evaluate $\int_1^2 \frac{2x + 1}{x^2(x + 1)}\, dx$. **[7 marks]**

Total 20 marks

Section B (Module 2)

Answer BOTH questions.

3. (a) (i) Use the fact that $\frac{1}{r} - \frac{1}{r + 1} = \frac{1}{r(r + 1)}$ to show that

$$S_n = \sum_{r=1}^{n} \left(\frac{1}{r(r + 1)}\right) = 1 - \frac{1}{n + 1}.$$ **[5 marks]**

(ii) Deduce, that as $n \to \infty$, $S_n \to 1$. **[1 mark]**

(b) The common ratio, r, of a geometric series is given by $r = \frac{5x}{4 + x^2}$. Find ALL the values of x for which the series converges. **[10 marks]**

(c) By substituting suitable values of x on both sides of the expansion of

$$(1 + x)^n = \sum_{r=0}^{n} {}^nC_r\, x^r,$$

show that

(i) $\sum_{r=0}^{n} {}^nC_r = 2^n$ **[2 marks]**

(ii) $\sum_{r=0}^{n} {}^nC_r\, (-1)^r = 0.$ **[2 marks]**

Total 20 marks

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4. The function, f, is given by $f(x) = 6 - 4x - x^3$.

(a) Show that

(i) f is everywhere strictly decreasing [4 marks]

(ii) the equation $f(x) = 0$ has a real root, α, in the closed interval $[1, 2]$ [4 marks]

(iii) α is the only real root of the equation $f(x) = 0$. [4 marks]

(b) If x_n is the n$^{\text{th}}$ approximation to α, use the Newton-Raphson method to show that the $(n + 1)^{\text{st}}$ approximation x_{n+1} is given by

$$x_{n+1} = \frac{2x_n^3 + 6}{3x_n^2 + 4}.$$ [8 marks]

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

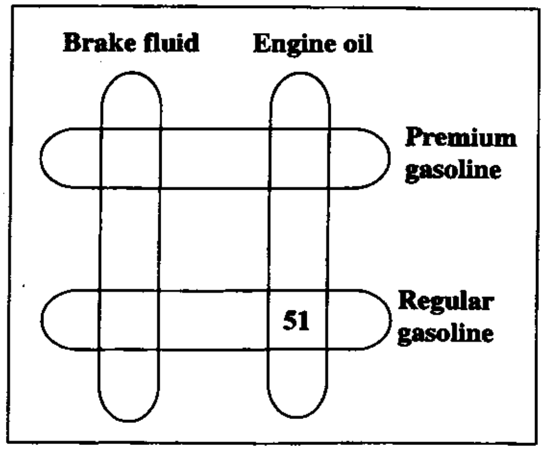
5. (a) On a particular day, a certain fuel service station offered 100 customers who purchased premium or regular gasoline, a free check of the engine oil or brake fluid in their vehicles. The services required by these customers were as follows:

15% of the customers purchased premium gasoline, the others purchased regular gasoline.

20% of the customers who purchased premium gasoline requested a check for brake fluid, the others requested a check for engine oil.

51 of the customers who purchased regular gasoline requested a check for engine oil, the others requested a check for brake fluid.

(i) Copy and complete the diagram below to represent the event space.



[3 marks]

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(ii) Find the probability that a customer chosen at random

a) who had purchased premium gasoline requested a check for engine oil

b) who had requested a check of the brake fluid purchased regular gasoline

c) who had requested a check of the engine oil purchased regular gasoline. **[6 marks]**

(b) A bag contains 12 red balls, 8 blue balls and 4 white balls. Three balls are drawn from the bag at random **without replacement.**

Calculate

(i) the total number of ways of choosing the three balls **[3 marks]**

(ii) the probability that ONE ball of EACH colour is drawn **[3 marks]**

(iii) the probability that ALL THREE balls drawn are of the SAME colour. **[5 marks]**

Total 20 marks

6. (a) Find the values of x for which

$$\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 2 & 1 & x \end{vmatrix} = 0.$$

[10 marks]

(b) Twelve hundred people visited an exhibition on its opening day. Thereafter, the attendance fell each day by 4% of the number on the previous day.

(i) Obtain an expression for the number of visitors on the n^{th} day. **[2 marks]**

(ii) Find the total number of visitors for the first n days. **[3 marks]**

(iii) The exhibition closed after 10 days. Determine how many people visited during the period for which it was opened. **[3 marks]**

(iv) If the exhibition had been kept opened indefinitely, what would be the maximum number of visitors? **[2 marks]**

Total 20 marks

END OF TEST

FORM TP 2005257

TEST CODE **02234032**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

$1\frac{1}{2}$ hours

23 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 1 question.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination paper consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer this question.

1. Table 1 presents data obtained from a biological investigation that involves two variables x and y.

Table 1

x	20	30	40	50
y	890	1640	2500	3700

It is believed that x and y are related by the formula, $y = bx^n$.

(a) (i) By taking logarithms to base 10 of both sides, convert $y = bx^n$ to the form $Y = nX + d$ where n and d are constants. [4 marks]

(ii) Hence, express

a) Y in terms of y

b) X in terms of x

c) d in terms of b. [3 marks]

(b) Use the data from Table 1 to complete Table 2.

Table 2

$\log_{10}x$	1.30		1.60	
$\log_{10}y$		3.21		3.57

[2 marks]

(c) In the graph on page 3, $\log_{10}x$ is plotted against $\log_{10}y$ for $1.3 \leq x \leq 1.7$.

(i) Assuming that the 'best straight line' is drawn to fit the data, determine

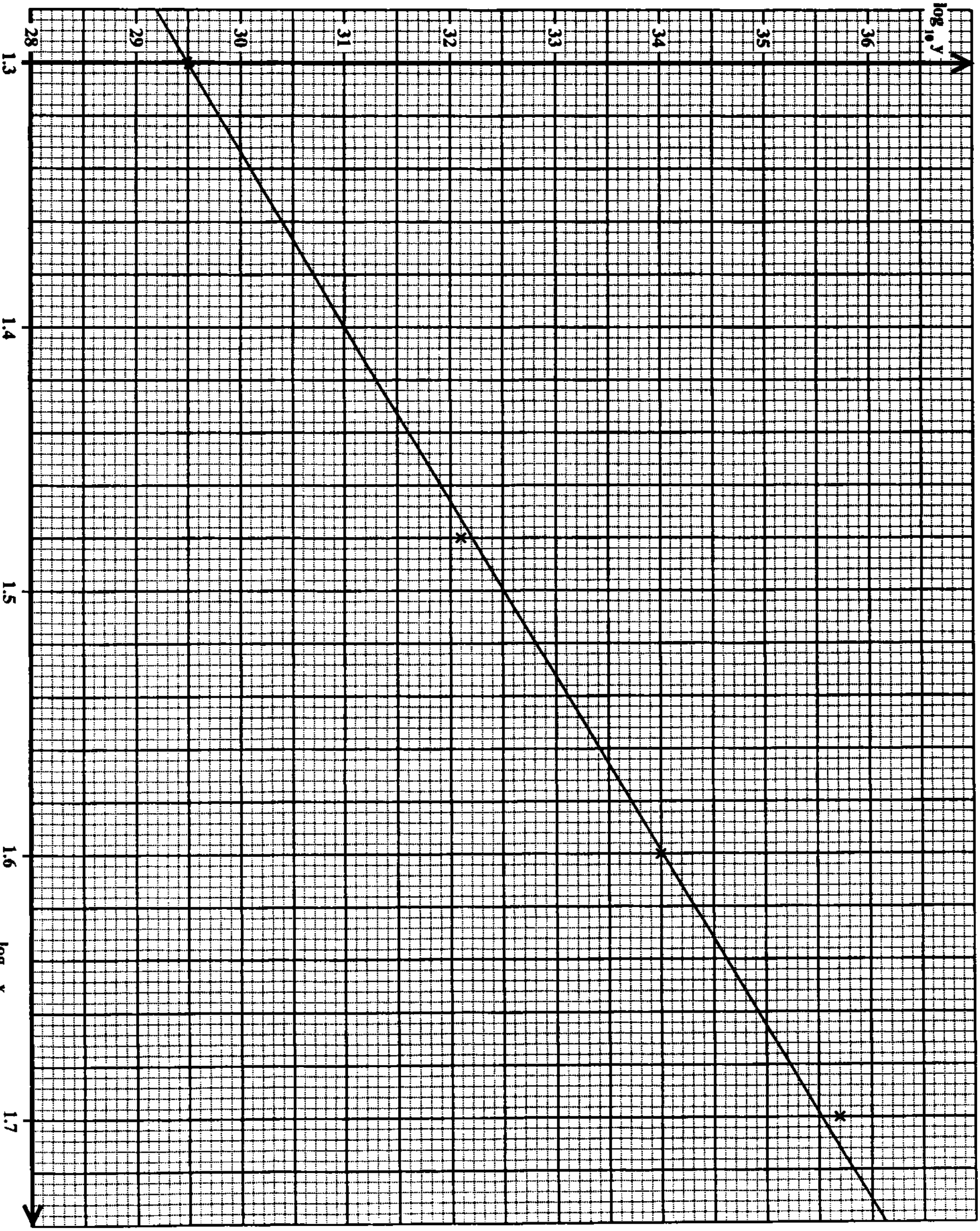
a) the gradient of this line [2 marks]

b) the value of b given that this line passes through (0, 1) [4 marks]

c) the value of each of the constants, n and d, in Part (a) (i) above. [2 marks]

(ii) Using the graph, or otherwise, estimate the value of x for which y is 1800. [3 marks]

Total 20 marks



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Section B (Module 2)

Answer this question.

2. Mr John Slick takes out an investment with an investment company which requires making **a fixed payment of \$A at the beginning of each year**. At the end of the investment period, John expects to receive a payout sum of money which is equal to the total payments made, together with interest added at the end of EACH year at a rate of r % per annum of the total sum in the fund.

The table below shows information on Mr Slick's investment for the first three years.

Year	Amount at Beginning of Year (\$)	Interest (\$)	Payout Sum \$
1	A	$A \times \frac{r}{100}$	$A + \left(A \times \frac{r}{100}\right)$ $= A\left(1 + \frac{r}{100}\right)$ $= AR$
2	A + AR	$(A + AR) \times \frac{r}{100}$	$(A + AR) + \left[(A + AR) \times \frac{r}{100}\right]$ $= (A + AR)\left(1 + \frac{r}{100}\right)$ $= (A + AR)R$ $= AR + AR^2$
3	$A + AR + AR^2$	$(A + AR + AR^2) \times \frac{r}{100}$	$(A + AR + AR^2)R$ $= AR + AR^2 + AR^3$

(a) Write expressions for

(i) the amounts at the beginning of Years 4 and 5 [2 marks]

(ii) payout sums at the end of Years 4 and 5. [2 marks]

(b) By using the information in the Table, or otherwise, write an expression for the amount at the beginning of the nth year. [2 marks]

(c) Show that the payout sum in (b) above is $\frac{\$\,AR\,(R^n - 1)}{R - 1}$ for $R > 1$. [7 marks]

(d) Find the value of A, **to the nearest dollar**, when $n = 20$, $r = 5$ and the payout sum in (c) above is \$500 000.00. [7 marks]

Total 20 marks

GO ON TO THE NEXT PAGE

Section C (Module 3)

Answer this question.

3. The output 3 x 1 matrix Y in a testing process in a chemical plant is related to the input 3 x 1 matrix X by means of the equation Y = AX, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}.$$

(a) Show that A is non-singular. **[5 marks]**

(b) Show that $X = A^{-1}Y$. **[3 marks]**

(c) Find A^{-1}. **[9 marks]**

(d) Find the input matrix X corresponding to the output matrix $Y = \begin{pmatrix} 19 \\ 34 \\ 42 \end{pmatrix}$. **[3 marks]**

Total 20 marks

END OF TEST

TEST CODE **02234010**

FORM TP 2006260

MAY/JUNE 2006

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 01

2 hours

22 MAY 2006 (a.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. Solve, for x, the equations

(a) $\log_2 x + \dfrac{2}{\log_2 x} = 3, x > 0$ **[5 marks]**

(b) $3^x = 5^{x-1}$. **[3 marks]**

Total 8 marks

2. Differentiate with respect to x the following:

(a) $y = e^{2x + \sin x}$ **[3 marks]**

(b) $y = \tan 3x + ln\,(x^2 + 4)$ **[4 marks]**

Total 7 marks

3. (a) Find the gradient of the curve $x^2 + xy = 2y^2$ at the point $P\,(-2, 1)$. **[5 marks]**

(b) Hence, find the equation of the normal to the curve at P. **[3 marks]**

Total 8 marks

4. If $y = \sin 2x + \cos 2x$,

(a) find $\dfrac{dy}{dx}$ **[3 marks]**

(b) show that $\dfrac{d^2y}{dx^2} + 4y = 0$. **[4 marks]**

Total 7 marks

5. Use the substitution indicated in EACH case to find the following integrals:

(a) $\int \sin^8 x \cos x \, dx$; $u = \sin x$ **[4 marks]**

(b) $\int x\sqrt{2x+1}\ dx$; $u^2 = 2x + 1$ **[6 marks]**

Total 10 marks

GO ON TO THE NEXT PAGE

Section B (Module 2)

Answer ALL questions.

6. A sequence $\{u_n\}$ of real numbers satisfies $u_{n+1}u_n = 3(-1)^n$; $u_1 = 1$.

   (a) Show that

   (i) $u_{n+2} = -u_n$ **[3 marks]**

   (ii) $u_{n+4} = u_n$. **[1 mark]**

   (b) Write the FIRST FOUR terms of this sequence. **[3 marks]**

Total 7 marks

7. (a) Verify that the sum, S_n, of the series $\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \ldots$, to n terms, is $S_n = \frac{2}{3}\left(1 - \frac{1}{2^{2n}}\right)$. **[4 marks]**

   (b) Three consecutive terms, $x - d$, x and $x + d$, $d > 0$, of an arithmetic series have sum 21 and product 315. Find the value of

   (i) x **[2 marks]**

   (ii) the common difference d. **[4 marks]**

Total 10 marks

8. If ${}^{(x-2)}C_2 = \frac{5}{2}\left({}^4C_3\right)$, $x > 3$,

   (a) show that $x^2 - 5x - 14 = 0$ **[4 marks]**

   (b) find x. **[2 marks]**

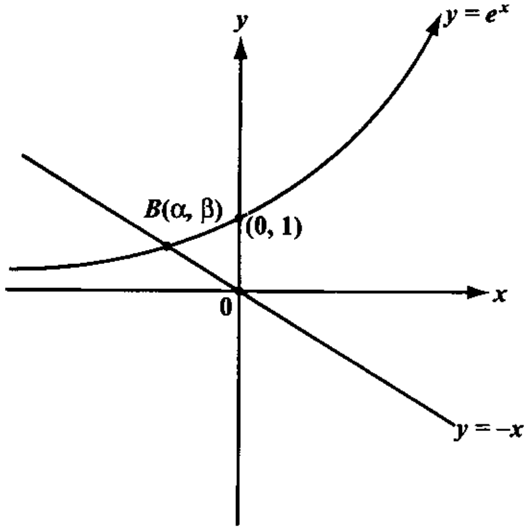
Total 6 marks

9. (a) Expand $(1 + ux)(2 - x)^3$ in powers of x up to the term in x^2, $u \in \mathbf{R}$. **[6 marks]**

   (b) Given that the coefficient of the term in x^2 is zero, find the value of u. **[2 marks]**

Total 8 marks

10.



The diagram above (**not drawn to scale**) shows the graphs of the two functions $y = e^x$ and $y = -x$.

(a) State the equation in x that is satisfied at B (α, β), the point of intersection of the two graphs. **[2 marks]**

(b) Show that α lies in the closed interval $[-1, 0]$. **[7 marks]**

Total 9 marks

Section C (Module 3)

Answer ALL questions.

11. A committee of 4 people is to be selected from a group consisting of 8 males and 4 females. Determine the number of ways in which the committee may be formed if it is to contain

(a) NO females **[2 marks]**

(b) EXACTLY one female **[3 marks]**

(c) AT LEAST one female. **[4 marks]**

Total 9 marks

12. (a) The letters H, R, D, S and T are consonants. In how many ways can the letters of the word HARDEST be arranged so that

(i) the first letter is a consonant? **[3 marks]**

(ii) the first and last letters are consonants? **[3 marks]**

(b) Find the probability that the event in (a) (i) above occurs. **[2 marks]**

Total 8 marks

13. The determinant Δ is given by

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Show that $\Delta = 0$ for any a, b and $c \in \mathbf{R}$. **[6 marks]**

Total 6 marks

14. (a) Write the following system of equations in the form $AX = D$.

$$\begin{aligned} x + y - z &= 2 \\ 2x - y + z &= 1 \\ 3x + 2z \quad &= 1 \end{aligned}$$

[2 marks]

(b) (i) Find the matrix B, the matrix of cofactors of the matrix A. **[5 marks]**

(ii) Calculate $\mathrm{B^T A}$. **[2 marks]**

(iii) Deduce the value of $|\mathrm{A}|$. **[1 mark]**

Total 10 marks

GO ON TO THE NEXT PAGE

15. A closed cylinder has a fixed height, h cm, but its radius, r cm, is increasing at the rate of 1.5 cm per second.

(a) Write down a differential equation for r with respect to time t secs. **[1 mark]**

(b) Find, in terms of π, the rate of increase with respect to time t of the total surface area, A, of the cylinder when the radius is 4 cm and the height is 10 cm. **[6 marks]**

$$[\, A = 2\pi r^2 + 2\pi rh]$$

Total 7 marks

END OF TEST

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

31 MAY 2006 (p.m)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) If $f(x) = x^3 \ln^2 x$, show that

(i) $f'(x) = x^2 \ln x(3 \ln x + 2)$

[5 marks]

(ii) $f''(x) = 6x \ln^2 x + 10x \ln x + 2x$.

[5 marks]

(b) The enrolment pattern of membership of a country club follows an exponential logistic function N,

$$N = \frac{800}{1 + ke^{-rt}}, k \in \mathbf{R}, r \in \mathbf{R},$$

where N is the number of members enrolled t years after the formation of the club. The initial membership was 50 persons and after one year, there are 200 persons enrolled in the club.

(i) What is the LARGEST number reached by the membership of the club?

[2 marks]

(ii) Calculate the EXACT value of k and of r. [6 marks]

(iii) How many members will there be in the club 3 years after its formation?

[2 marks]

Total 20 marks

2. (a) (i) Express $\dfrac{1 + x}{(x - 1)(x^2 + 1)}$ in partial fractions. [6 marks]

(ii) Hence, find $\displaystyle\int \frac{1 + x}{(x - 1)(x^2 + 1)}\, dx$. [3 marks]

(b) Given that $I_n = \displaystyle\int_0^1 x^n e^x\, dx$, where $n \in N$.

(i) Evaluate I_1. [4 marks]

(ii) Show that $I_n = e - nI_{n-1}$. [4 marks]

(iii) Hence, or otherwise, evaluate I_3, writing your answer in terms of e.

[3 marks]

Total 20 marks

GO ON TO THE NEXT PAGE

Section B (Module 2)

Answer BOTH questions.

3. (a) (i) Show that the terms of

$$\sum_{r=1}^{m} \ln 3^r$$

are in arithmetic progression. **[3 marks]**

(ii) Find the sum of the first 20 terms of this series. **[4 marks]**

(iii) Hence, show that $\sum_{r=1}^{2m} \ln 3^r = (2m^2 + m) \ln 3$. **[3 marks]**

(b) The sequence of positive terms, $\{x_n\}$, is defined by $x_{n+1} = x_n^2 + \frac{1}{4}$, $x_1 < \frac{1}{2}$.

(i) Show, by mathematical induction, or otherwise, that $x_n < \frac{1}{2}$ for all positive integers n. **[7 marks]**

(ii) By considering $x_{n+1} - x_n$, or otherwise, show that $x_n < x_{n+1}$. **[3 marks]**

Total 20 marks

4. (a) Sketch the functions $y = \sin x$ and $y = x^2$ on the SAME axes. **[5 marks]**

(b) Deduce that the function $f(x) = \sin x - x^2$ has EXACTLY two real roots. **[3 marks]**

(c) Find the interval in which the non-zero root α of $f(x)$ lies. **[4 marks]**

(d) Starting with a first approximation of α at $x_1 = 0.7$, use one iteration of the Newton-Raphson method to obtain a better approximation of α to 3 decimal places. **[8 marks]**

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

5. (a) (i) How many numbers lying between 3 000 and 6 000 can be formed from the digits, 1, 2, 3, 4, 5, 6, if no digit is used more than once in forming the number? **[5 marks]**

(ii) Determine the probability that a number in 5 (a) (i) above is even. **[5 marks]**

(b) In an experiment, p is the probability of success and q is the probability of failure in a single trial. For n trials, the probability of x successes and $(n - x)$ failures is represented by ${}^{n}C_{x}p^{x}q^{n-x}$, $n > 0$. Apply this model to the following problem.

The probability that John will hit the target at a firing practice is $\frac{5}{6}$. He fires 9 shots. Calculate the probability that he will hit the target

(i) AT LEAST 8 times **[7 marks]**

(ii) NO MORE than seven times. **[3 marks]**

Total 20 marks

6. (a) If $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 2 & 1 \\ 1 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(i) find AB **[3 marks]**

(ii) deduce A^{-1}. **[3 marks]**

(b) A nursery sells three brands of grass-seed mix, P, Q and R. Each brand is made from three types of grass, C, Z and B. The number of kilograms of each type of grass in a bag of each brand is summarised in the table below.

Grass Seed Mix	**Type of Grass (Kilograms)**		
	***C*-grass**	***Z*-grass**	***B*-grass**
Brand P	2	2	6
Brand Q	4	2	4
Brand R	0	6	4
Blend	c	z	b

A blend is produced by mixing p bags of Brand P, q bags of Brand Q and r bags of Brand R.

(i) Write down an expression in terms of p, q and r, for the number of kilograms of Z-grass in the blend. **[1 mark]**

(ii) Let c, z and b represent the number of kilograms of C–grass, Z–grass and B–grass respectively in the blend. Write down a set of THREE equations in p, q, r, to represent the number of kilograms of EACH type of grass in the blend. **[3 marks]**

(iii) Rewrite the set of THREE equations in (b) (ii) above in the matrix form $MX = D$ where M is a 3 by 3 matrix, X and D are column matrices. **[3 marks]**

(iv) Given that M^{-1} exists, write X in terms of M^{-1} and D. **[3 marks]**

(v) Given that $M^{-1} = \begin{pmatrix} -0.2 & -0.2 & 0.3 \\ 0.35 & 0.1 & -0.15 \\ -0.05 & 0.2 & -0.05 \end{pmatrix}$,

calculate how many bags of EACH brand, P, Q, and R, are required to produce a blend containing 30 kilograms of C-grass, 30 kilograms of Z-grass and 50 kilograms of B-grass. **[4 marks]**

Total 20 marks

END OF TEST

FORM TP 2006262

TEST CODE **02234032**

MAY/JUNE 2006

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

$1\frac{1}{2}$ hours

22 MAY 2006 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 1 question.
The maximum mark for each section is 20.
The maximum mark for this examination is 60.
This examination paper consists of 4 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer this question.

1. The rate of increase of the number of algae with respect to time, t days, is equal to k times $f(t)$, where $f(t)$ is the number of algae at any given time t and $k \in \mathbf{R}$.

(a) Obtain a differential equation involving $f(t)$ which may be used to model this situation. **[1 mark]**

(b) Given that

- the number of algae at the beginning is 10^6
- the number of algae doubles every 2 days,

(i) determine the values of $f(0)$ and $f(2)$ **[2 marks]**

(ii) show that

a) $k = \frac{1}{2} \ln 2$ **[10 marks]**

b) $f(t) = 10^6(2^{t/2})$ **[5 marks]**

(iii) determine the approximate number of algae present after 7 days. **[2 marks]**

Total 20 marks

Section B (Module 2)

Answer this question.

2. (a) A car was purchased at the beginning of the year, for ***P* dollars**. The value of a car **at the end of each year** is estimated to be the value **at the beginning of the year** multiplied by $(1 - \frac{1}{q})$, $q \in N$.

(i) Copy and complete the table below showing the value of the car for the first five years after purchase.

	Year 1	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Value at the Beginning of Year ($)	P	$P(1 - \frac{1}{q})$	$P(1 - \frac{1}{q})^2$		
Value at the End of Year ($)	$P(1 - \frac{1}{q})$	$(1 - \frac{1}{q})\left[P(1 - \frac{1}{q})\right]$ $= P(1 - \frac{1}{q})^2$			

[3 marks]

(ii) Describe FULLY the sequence shown in the table. **[2 marks]**

(iii) Determine, in terms of P and q, the value of the car n years after purchase. **[1 mark]**

(b) If the original value of the car was \$20 000.00 and the value at the end of the fourth year was \$8 192.00, find

(i) the value of q **[5 marks]**

(ii) the estimated value of the car after five years **[2 marks]**

(iii) the LEAST integral value of n, the number of years after purchase, for which the estimated value of the car falls below \$500.00. **[7 marks]**

Total 20 marks

Section C (Module 3)

Answer this question.

3. (a) A box contains 8 green balls and 6 red balls. Five balls are selected at random. Find the probability that

(i) ALL 5 balls are green [4 marks]

(ii) EXACTLY 3 of the five balls are red [4 marks]

(iii) at LEAST ONE of the five balls is red. [3 marks]

(b) Use the method of row reduction to echelon form on the augmented matrix for the following system of equations to show that the system is inconsistent. [9 marks]

$$\begin{aligned} x + 2y + 4z &= 6 \\ y + 2z &= 3 \\ x + y + 2z &= 1 \end{aligned}$$

Total 20 marks

END OF TEST

TEST CODE **02234010/SPEC**

FORM 02234010/SPEC2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

Paper 01

90 minutes

READ THE FOLLOWING DIRECTIONS CAREFULLY

1. In addition to this test booklet, you should have an answer sheet.
2. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
3. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

   Sample Item

   The expression $(1+\sqrt{3})^2$ is equivalent to

   (A) 4
   (B) 10
   (C) $1+3\sqrt{3}$
   (D) $4+2\sqrt{3}$

   Sample Answer

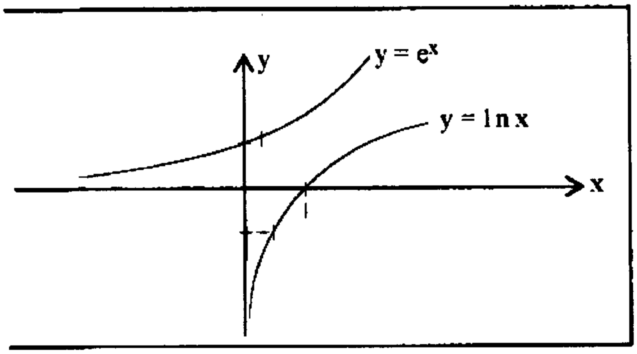
   Ⓐ Ⓑ Ⓒ ●

   The best answer to this item is "$4+2\sqrt{3}$", so answer space (D) has been blackened.

4. If you want to change your answer, be sure to erase your old answer completely and fill in your new choice.
5. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can come back to the harder item later.
6. You may do any rough work in this booklet.
7. The use of non-programmable calculators is allowed.
8. This test consists of 45 items. You will have 90 minutes to answer them.
9. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

<u>Item 1</u> refers to the following graph.

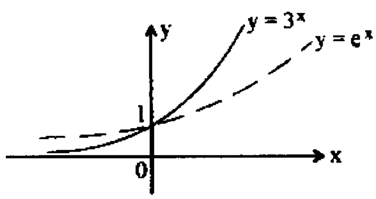


1. The graphs of $y = e^x$ and $y = \ln x$ in the diagram above are reflections of each other in the line

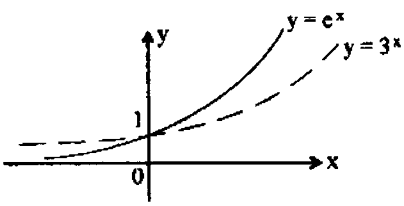
(A) $y = 0$
(B) $x = 0$
(C) $y = -x$
(D) $y = x$

2. Which of the graphs in the diagrams below represents $y = 3^x$ and $y = e^x$?

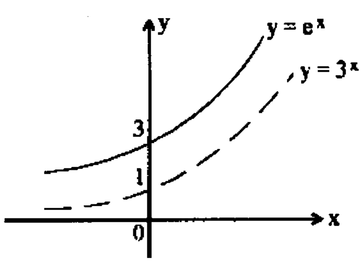
(A)



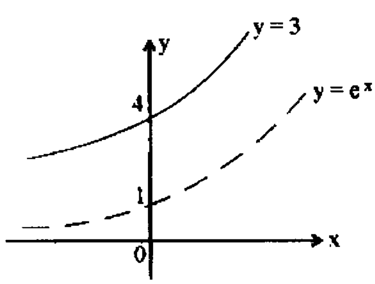
(B)



(C)



(D)



3. $\frac{1}{2}\ln 4 - \frac{1}{2}\ln 3$ can also be expressed as

I. $\frac{1}{2}\ln 1$

II. $\frac{1}{2}\ln\frac{4}{3}$

III. $\ln 2 - \ln\sqrt{3}$

IV. $\ln\frac{2}{\sqrt{3}}$

(A) I and II only
(B) I, II and III only
(C) II, III and IV only
(D) III and IV only

4. If $5^x = 20$, then $x =$

(A) $\log\left(\frac{5}{20}\right)$

(B) $\log\left(\frac{20}{5}\right)$

(C) $\frac{\log 5}{\log 20}$

(D) $\frac{\log 20}{\log 5}$

5. $\log_c\left(\frac{a^2}{b}\right)$ can be written as

(A) $2\log_c a - \log_c b$

(B) $2\log_c\left(\frac{a}{b}\right)$

(C) $2\log_c\left(\frac{b}{a}\right)$

(D) $2\log_c b^a$

6. $\frac{d}{dx}\left(e^{3x^2+2x+1}\right)$ is

(A) $(6x+2)\, e^{6x-2}$
(B) $(6x+2)\, e^{3x^2+2x+1}$
(C) $(3x^2+2x+1)\, e^{6x-2}$
(D) $(3x^2+2x+1)\, e^{3x^2+2x+1}$

7. A curve is defined parametrically by the equations $x = t^2$, $y = t(1 - t^2)$. The gradient of the curve, in terms of t, is

(A) $\frac{2t}{1-3t^2}$
(B) $\frac{1-3t^2}{2t}$
(C) $2t(1 - 2t)$
(D) $2t(1 + 2t)$

8. For $x^3y - 3 = -6x$, $\frac{dy}{dx}$ at the point where $x = 1$ and $y = -3$ is equal to

(A) -15
(B) $\frac{2}{3}$
(C) 3
(D) 11

9. Given $y = \ln (2x+3)^3$, then $\frac{dy}{dx}$ is

(A) $\frac{2x}{2x+3}$
(B) $\frac{2}{2x+3}$
(C) $\frac{6x}{2x+3}$
(D) $\frac{6}{2x+3}$

10. If the function $f(x)$ is defined by $f(x) = \cos x$ then $f''(x)$ is

(A) - cos x
(B) - sin x
(C) cos x
(D) sin x

11. The partial fractions expression for $\frac{5}{(x+2)(x-3)}$ may be written as

(A) $\frac{1}{x+2} + \frac{1}{(x-3)}$
(B) $\frac{-1}{x+2} + \frac{1}{x-3}$
(C) $\frac{1}{x+2} + \frac{-1}{x-3}$
(D) $\frac{-1}{x+2} + \frac{-1}{x-3}$

12. Which of the following functions, when integrated w.r.t. x, gives the result $x - \ln x^2 + K$?

(A) $\frac{1}{1-x^2}$
(B) $\frac{1-2x}{x^2}$
(C) $\frac{x-2}{x}$
(D) $1 - \frac{2}{x^2}$

GO ON TO THE NEXT PAGE

13. $\int \cos^2 x \, dx$ is equal to

(A) $\frac{1}{2} \sin^2 x + c$

(B) $\frac{1}{3} \cos^3 x + c$

(C) $\frac{1}{4} \sin 2x + c$

(D) $\frac{1}{4} \sin 2x + \frac{1}{2} x + c$

14. $\int \frac{x}{x^2+3} \, dx$ is equal to

(A) $\frac{1}{2} \ln (x^2 + 3) + c$
(B) $2 \ln (x^2 + 3) + c$
(C) $2x \ln (x^2 + 3) + c$
(D) $(x^2 + 3) \ln x + c$

15. $\int xe^{2x} dx$ may be expressed as

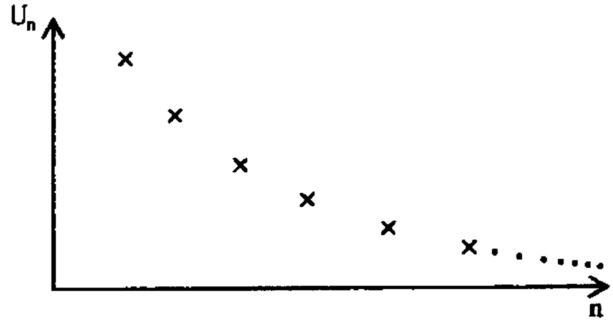
(A) $2xe^{2x} + e^{2x} + c$

(B) $2xe^{2x} - 4e^{2x} + c$

(C) $\frac{1}{2} xe^{2x} - \frac{1}{4} e^{2x} + c$

(D) $\frac{1}{2} xe^{2x} + \frac{1}{2} x^2 e^{2x} + c$

<u>Item 16</u> refers to the diagram below.



16. The term that best describes the behaviour of the sequence $\{U_n\}$ shown above is

(A) periodic
(B) finite
(C) divergent
(D) convergent

GO ON TO THE NEXT PAGE

Items **17 - 19** refer to S as defined below.

$S = 2^0 - 2^1 + 2^2 - 2^3 + 2^4$

17. S is best described as a

(A) finite series
(B) infinite series
(C) finite sequence
(D) infinite sequence

18. The general term in S is best defined by

(A) $(-1)^r \, 2^r$
(B) $(-1)^r \, (-2)^r$
(C) $(-1)^{r-1} \, (2)^r$
(D) $(-1)^{r-1} \, (-2)^{r-1}$

19. S may be written as

(A) $\sum_{r=0}^{r}(-1)^r(2^r)$

(B) $\sum_{r=1}^{r}(-1)^r(2)^{r-1}$

(C) $\sum_{r=0}^{4}(-1)^r(2)^r$

(D) $\sum_{r=1}^{4}(-1)^{r-1}(-2)^{r-1}$

Items **20 - 22** refer to E as defined below.

$$E = \sum_{n=19}^{30} \frac{3^n}{n}$$

20. The number of terms in the expansion of E is

(A) 10
(B) 11
(C) 12
(D) 13

21. The r^{th} term of E is 3^{24}. The value of r is

(A) 6
(B) 8
(C) 9
(D) 10

22. Which of the series below has its n^{th} term equal to $\frac{a}{2^n}$?

(A) $\sum_{r=1}^{n}\frac{a}{2^{-r}}$

(B) $\sum_{r=1}^{n}\frac{a}{2^r}$

(C) $\sum_{r=1}^{n}\frac{a}{2^{r-1}}$

(D) $\sum_{r=0}^{n}\frac{a}{2^{r-1}}$

23. $\sum_{r=1}^{20}(10-2r)=$

(A) -1410
(B) -220
(C) -10
(D) 220

24. The first term of an AP is 'a' and its common difference is -1. The sum of the first 10 terms is equal to

(A) 5(2a - 9)

(B) 5(2a + 9)

(C) 10(2a + 11)

(D) 10(2a - 11)

25. The second and fifth terms of a convergent geometric series with first term $\frac{81}{2}$ are 27 and 8 respectively. The sum to infinity of this series is

(A) $\frac{2}{3}$

(B) $\frac{243}{2}$

(C) $\frac{27}{2}$

(D) $\frac{81}{2}$

26. The sum to infinity of the geometric series $a + a^2 + a^3 + \ldots$ is $4a$ $(a \neq 0)$. The common ratio is

(A) $\frac{-3}{4}$

(B) $\frac{3}{4}$

(C) $\frac{4}{3}$

(D) $\frac{5}{4}$

27. ${}^{n}C_{r-1} =$

(A) $\frac{n!}{(n-r-1)!}$

(B) $\frac{n!}{(n-r+1)!}$

(C) $\frac{n!}{(n-r+1)!\,(r-1)!}$

(D) $\frac{n!}{[n-(r-1)]!\,r!}$

28. The 3rd term in the expansion of $(2 - \frac{x}{2})^6$ in ascending powers of x is

(A) $-60x^2$
(B) $-20x^3$
(C) $60x^2$
(D) $20x^3$

GO ON TO THE NEXT PAGE

29. The coefficient of x^2 in the expansion of $(2 - 3x)^5$ is

(A) -720
(B) -240
(C) 240
(D) 720

30. The function $f(x) = x^3 - 3x - 3$ has a root in the closed interval

(A) [-10, -8]
(B) [-2, 0]
(C) [2, 3]
(D) [5, 6]

31. The number of ways in which a committee of four men and six women can be seated in a row if they can sit in any position is

(A) 2!
(B) 4!
(C) 6!
(D) 10!

32. The number of ways in which 3 boys and 2 girls can sit so that no two persons of the same sex sit next to each other is

(A) 3 x 2
(B) 3! + 2!
(C) 3! x 2!
(D) 5!

33. A team of eleven players is to be chosen from a squad of 16 players. Given that 2 players must be chosen, the number of ways in which the team can be chosen is

(A) $^{14}C_{11}$
(B) $^{14}C_{9}$
(C) $^{16}C_{11}$
(D) $^{16}C_{9}$

34. In how many ways can two persons be selected from a group of 10 persons?

(A) 20
(B) 45
(C) 90
(D) 100

35. What is the probability that an integer chosen at random from 1, 2, 3, 4, 5, 6, 7, 9, 11, 15 is prime?

(A) $\frac{3}{10}$

(B) $\frac{4}{10}$

(C) $\frac{5}{10}$

(D) $\frac{6}{10}$

36. The letters P, Q, R, S, T and U are arranged randomly in a line. What is the probability that P and Q are next to each other?

(A) $\frac{2 \times 5!}{6!}$

(B) $\frac{2 \times 6!}{5!}$

(C) $\frac{5 \times 2!}{6!}$

(D) $\frac{5 \times 6!}{2!}$

37. What is the probability that an integer chosen at random from 1, 2, 3, 4, 5, 6, 7, 9 11, 15 is divisible by 3?

(A) $\frac{1}{10}$

(B) $\frac{2}{10}$

(C) $\frac{3}{10}$

(D) $\frac{4}{10}$

38. A and B are two independent events. Given P (A) = 0.3 and P (B) = 0.4, which of the following are true?

I. $P(A \cup B) = 0.7$
II. $P(A \cap B) = 0.12$
III. $P(A \mid B) = 0.3$
IV. $P(A \mid \overline{B}) = 0.4$

(A) I and II only
(B) II and III only
(C) II and IV only
(D) III and IV only

39. A fair die is tossed twice. What is the probability that at least one toss results in a 5?

(A) $\frac{2}{36}$

(B) $\frac{10}{36}$

(C) $\frac{11}{36}$

(D) $\frac{25}{36}$

Items 40 refers to the matrices P and Q below.

$$P = \begin{bmatrix} a & b & c \end{bmatrix}, Q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

40. The product PQ is

(A) [ax + by + cz]

(B) $\begin{bmatrix} ax \\ by \\ cz \end{bmatrix}$

(C) [ax by cz]

(D) not possible

Items 41 - 42 refer to the matrix $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$

41. The cofactor of the circled element, -2, is

(A) -2
(B) -1
(C) 0
(D) 2

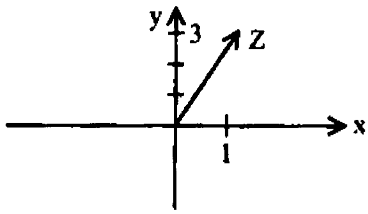
42. The determinant of the given matrix is

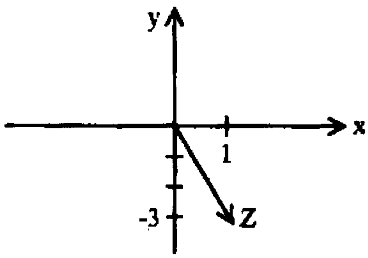
(A) -5
(B) -3
(C) 3
(D) 5

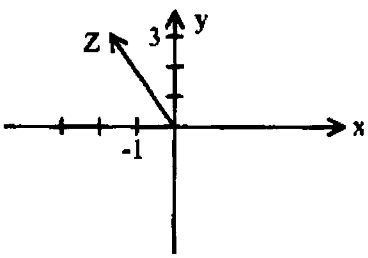
43. If z is the complex number 2 - i, then z^2 equals

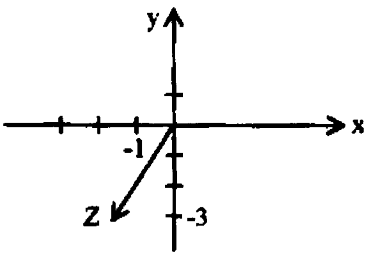
(A) 3
(B) 4
(C) 3 - 4i
(D) 4 - 3i

44. Which Agrand diagram best represents the complex number $z = 1 - i\sqrt{8}$?

(A) 

(B) 

(C) 

(D) 

45. Determine $\text{Im}\left(\frac{1}{z}\right)$, where $z = \frac{3 - i}{1 + i}$

(A) $-\frac{2}{5}$

(B) $-\frac{1}{5}$

(C) $\frac{1}{5}$

(D) $\frac{2}{5}$

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

FORM TP 02234020/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) (i) Using the fact that $e^{-x} = \frac{1}{e^x}$ or otherwise, show that,

$$\frac{d}{dx}(e^{-x}) = -e^{-x}.$$ **[2 marks]**

(ii) Hence, evaluate $\int x^2 e^{-x}\,dx$. **[4 marks]**

(b) (i) a) Find $\frac{dy}{dx}$ when $y = \tan^{-1}(3x)$. **[4 marks]**

b) Hence, find $\int \frac{(x+2)}{1+9x^2}\,dx$. **[4 marks]**

(ii) Show that if $y = \frac{\ln(5x)}{x^2}$ then $\frac{dy}{dx} = \frac{1-\ln(25x^2)}{x^3}$. **[6 marks]**

(c) Solve the first order differential equation

$$y\tan x\frac{dy}{dx} = (4+y^2)\sec^2 x.$$ **[5 marks]**

Total 25 marks

2 (a) In 1950, the world population was 2.5 billion and it grew to 5 billion in 1987. The world's population grows exponentially so that at time t years the population is $N = 2.5\,e^{kt}$ where t =0 corresponds to the year 1950 and N is measured in billions of people.

Find

(i) the <u>exact</u> value of k **[3 marks]**

(ii) the <u>exact</u> value of N in 2003 **[2 marks]**

(iii) the year in which N =10. **[5 marks]**

(b) Given that $y = u\cos 3x + v\sin 3x$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx}3y = -30\sin 3x,$$

Find

(i) the values of the constants u and v **[10 marks]**

(ii) the general solution of the differential equation. **[5 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Find constants A and B such that

$$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}.$$ **[5 marks]**

(ii) Hence, find the value of S where

$$S = \sum_{r=1}^{n} \frac{1}{2r-1)(2r+1)}.$$ **[5 marks]**

(iii) Deduce the sum to infinity of S. **[3 marks]**

(b) (i) Find the n^{th} term of the series 1(2) + 2(5) + 3(8) +… **[2 marks]**

(ii) Prove, by Mathematical Induction, that the sum to n terms of the series in (b)(i) above is $n^2(n + 1)$. **[10 marks]**

Total 25 marks

4. (a) Given the series $\frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + ...$

(i) show that the series is geometric **[4 marks]**

(ii) find the sum of the series to n terms. **[4 marks]**

(b) Use Maclaurin's Theorem to find the **first** three non-zero terms in the power series expansion of $\cos 2x$. **[7 marks]**

(c) (i) Expand up to and including the term in x^3

$$\sqrt{\left(\frac{1+x}{1-x}\right)},$$

stating the values of x for which the expansion is valid. **[6 marks]**

(ii) By taking $x = 0.02$ find an approximation for $\sqrt{51}$, correct to 5 decimal places. **[4 marks]**

Total 25 marks

SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Two cards are drawn without replacement from ten cards which are numbered 1 to 10. Find the probability that

(i) the numbers on **BOTH** cards are even **[4 marks]**

(ii) the number on one card is odd and the number on the other card is even. **[4 marks]**

(b) A journalist reporting on criminal cases classified 150 criminal cases by the age (in years) of the criminal and by the type of crime committed, violent or non-violent. The information is presented in the table below.

Type of Crime	Age (in years)		
	Less than 20	**20 to 39**	**40 or older**
Violent	27	41	14
Non-violent	12	34	22

What is the probability that a case randomly selected by the journalist

(i) a) is a violent crime? **[2 marks]**

b) was committed by someone LESS than 40 years old? **[4 marks]**

c) is a violent crime OR was committed by a person LESS than 20 years old? **[5 marks]**

d) is a violent crime that was committed by a person LESS than 20 years old? **[2 marks]**

(ii) Two criminal cases are randomly selected for review by a judge. What is the probability that BOTH cases are violent crimes? **[4 marks]**

Total 25 marks

6. (a) Solve the following equation using determinants:

$$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

[10 marks]

(b) Solve the following set of equations:

$$\begin{aligned} x_1 - 4x_2 - 2x_3 &= 21 \\ 2x_1 + x_2 + 2x_3 &= 3 \\ 3x_1 + 2x_2 - x_3 &= -2 \end{aligned}$$

[10 marks]

(c) (i) Express the complex number $\dfrac{4-2i}{1-3i}$ in the form of $a + bi$ where a and b are real numbers. **[4 marks]**

(ii) Show that the argument of the complex number in (c)(i) above is $\dfrac{\pi}{4}$.

[1 mark]

Total 25 marks

END OF TEST

FORM TP 02234032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03B

$1\frac{1}{2}$ ***hours***

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

The maximum mark for each question is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL THREE** questions.

3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

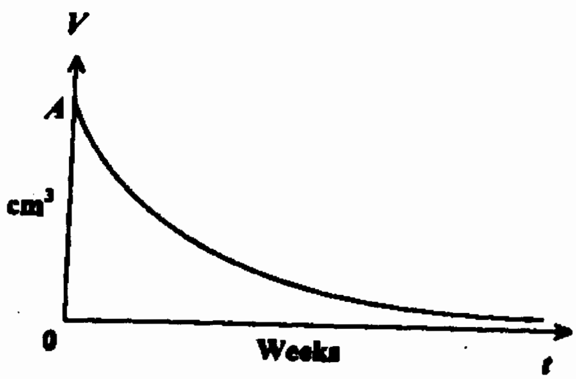
Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer this question.

1. The diagram below, **not drawn to scale,** shows the variation in the volume, V cm^3, of an air freshener block with time, t weeks.



The variation can be written as

$$\frac{dV}{dt} = -kV.$$

(a) Describe clearly in words the variation shown in the diagram above. **[2 marks]**

(b) Show that $V = Ae^{-kt}$, where A is a constant. **[5 marks]**

(c) Initially, an air freshener block has a volume of 64 cm^3. It loses half its volume after 6 weeks. Show that $V = 64e^{\left(\ln\frac{1}{2}\right)\frac{t}{6}}$. **[8 marks]**

(d) The air freshener block becomes ineffective when its volume reaches 6 cm^3. Calculate the time, to the NEAREST week, at which the block should be replaced. **[5 marks]**

Total 20 marks

SECTION B (MODULE 2)

Answer this question.

2. (a) The sum to infinity of a GP is 10 times the first term. Find the common ratio of the GP. **[3 marks]**

(b) A ball is projected vertically upwards to a height of 81 cm above a horizontal floor. It drops to the floor and bounces until it comes to a stop. After each bounce on the floor, the ball rises vertically to a height that is $\frac{2}{3}$ of the distance it dropped.

(i) Find

a) h_1, the distance the ball travels before the first bounce
b) h_2, the distance the ball travels before the second bounce
c) h_3, the distance the ball travels before the third bounce.

[3 marks]

(ii) Given that h_n, $n > 2$, is the distance the ball has travelled between the n^{th} bounce and the **previous** bounce, express

a) h_n in terms of h_{n-1}
b) h_{n-1} in terms of h_{n-2}. **[2 marks]**

(iii) Hence, show that

$$h_n = \left(\frac{2}{3}\right)^{n-1} h_1$$ **[3 marks]**

(iv) Show that the TOTAL distance that the ball has travelled just before the n^{th} bounce is

$$486\left(1-\left(\frac{2}{3}\right)^n\right) \text{ cm.}$$ **[6 marks]**

(v) Deduce that the TOTAL distance travelled by the ball when it stops bouncing is approximately 486 cm. **[3 marks]**

Total 20 marks

SECTION C (MODULE 3)

Answer this question.

3. (a) (i) A singer is scheduled to sing 6 particular songs without repetition at a cultural show. In how many ways can this singer's schedule be arranged? **[2 marks]**

(ii) If the singer has altogether 13 suitable songs, in how many ways can a schedule of 6 songs be prepared? **[3 marks]**

(b) A biology examination includes 4 True or False questions. The probability of a student guessing the correct answer to any question is $\frac{1}{2}$. Use the probability model

$$P(r) = \frac{n!}{r!(n-r)!}p^r q^{n-r}$$

where n is the number of questions
r is the number of observed successes
p is the probability of guessing correctly
q is the probability of guessing incorrectly

to answer the questions below.

What is the probability of a student guessing the correct answer to

(i) At LEAST ONE of the four questions correctly? **[5 marks]**

(ii) EXACTLY ONE of the four questions correctly? **[2 marks]**

(c) The transformation in three-dimensional space of a point, P, with coordinates (x, y, z) is represented below.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 9 \\ 1 \end{pmatrix}$$

By row-reducing the augmented matrix, find the coordinates of P. **[8 marks]**

Total 20 marks

02234010/CAPE/K - 2007

CARIBBEAN EXAMINATIONS COUNCIL
HEADQUARTERS

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

PAPER 01

KEY

CARIBBEAN EXAMINATIONS COUNCIL

Pure Mathematics Unit 2

Item	Key	Item	Key
1	D	24	A
2	A	25	B
3	C	26	B
4	D	27	C
5	A	28	C
6	B	29	D
7	B	30	C
8	C	31	D
9	D	32	C
10	A	33	B
11	B	34	B
12	C	35	D
13	D	36	A
14	A	37	D
15	C	38	B
16	D	39	C
17	A	40	A
18	A	41	A
19	C	42	C
20	C	43	C
21	C	44	B
22	B	45	D
23	B		

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

PAPER 02

SOLUTIONS
&
MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) (i) $\frac{d}{dx}\left(e^{-x}\right) = \frac{d}{dx}\left(\frac{1}{e^x}\right)$

$$= \frac{e^x(0)-(1)\left(e^x\right)}{\left(e^x\right)^2} \qquad \text{(1 mark)}$$

$$= \frac{-e^x}{\left(e^x\right)^2}$$

$$= \frac{-1}{e^x}$$

$$= -e^{-x} \qquad \text{(1 mark)}$$

[2 marks]

(ii) $\int x^2 e^{-x}\,dx$; let $u = x^2$ so that $du = 2x\,dx$, and

let $dv = e^{-x}dx$ so that $v = -e^{-x}$, then (1 mark)

$$\int x^2e^{-x}dx = -x^2e^{-x}+2\int e^{-x}x\,dx \qquad \text{(1 mark)}$$

$$= -x^2e^{-x}+2\left[-xe^{-x}+\int e^{-x}dx\right] \qquad \text{(1 mark)}$$

$$= -x^2e^{-x}-2xe^{-x}-2e^{-x}+c \qquad \text{(1 mark)}$$

i.e. $= -e^{-x}\left[x^2+2x+2\right]+c$ [4 marks]

(b) (i) a) $y = \tan^{-1}(3x) \Rightarrow \tan y = 3x$ (1 mark)

$$\Rightarrow \sec^2 y\,\frac{dy}{dx} = 3$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{\sec^2 y} \qquad \text{(1 mark)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+\tan^2 y} \qquad \text{(1 mark)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} \qquad \text{(1 mark)}$$

[4 marks]

b) $\int \frac{x+2}{1+9x^2}\,dx = \int \frac{2}{1+9x^2}\,dx + \int \frac{x}{1+9x^2}\,dx$ (1 mark)

$= \frac{2}{3}\tan^{-1}(3x) + \frac{1}{18}\ln(1+9x^2) + k$ (3 marks)

[4 marks]

(ii) $y = \frac{\ln(5x)}{x^2}$, Using quotient rule :

$\Rightarrow \frac{dy}{dx} = \frac{x^2\frac{1}{x} - (2x)\ln(5x)}{x^4}$ (3 marks)

$= \frac{1-2\ln(5x)}{x^3}$ (1 mark)

$= \frac{1-\ln(5x)^2}{x^3}$ (1 mark)

$= \frac{1-\ln(25x^2)}{x^3}$ (1 mark)

[6 marks]

(c) $y\tan x\,\frac{dy}{dx} = \left(4+y^2\right)\sec^2 x$

$\frac{y}{4+y^2}\frac{dy}{dx} = \frac{\sec^2 x}{\tan x}$ (1 mark)

$\frac{y\,dy}{4+y^2} = \frac{\sec^2 x\,dx}{\tan x}$ (1 mark)

$\int \frac{y\,dy}{4+y^2} = \int \frac{\sec^2 x\,dx}{\tan x}$ (1 mark)

$\frac{1}{2}\ln\left(4+y^2\right) = \ln\tan x + c$ (2 marks)

[5 marks]

Total 25 marks

Specific Objectives: (b) 1, 5, 6; (c) 5, 6, 7, 8, 9,10

Question 2

(a) $N = 2.5\, e^{kt}$ (given)

(i) In 1987, t = 37 and N = 5.0 (1 mark)

$\Rightarrow 5.0 = 2.5e^{k}$ (1 mark)

$\Rightarrow e^{37k} = 2 \Rightarrow k = \frac{1}{37} \ln 2$ (1 mark)

[3 marks]

(ii) In 2003, $t = 53 \Rightarrow N = 2.5 \times e^{\frac{53}{57}\ln 2} = 2.5(\ln 2)^{\frac{53}{37}}$ (2 marks)

[2 marks]

(iii) $N = 10 \Rightarrow \quad 10 = 2.5^{kt} \Rightarrow e^{kt} \Rightarrow e^{kt} = 4$ (2 marks)

$\Rightarrow \quad kt = \ln 4 \quad = 2 \ln 2$ (1 marks)

$\Rightarrow \quad t = \frac{2}{k}\ln 2 = 2 \times 37 \quad = 74$ (1 mark)

i.e. N = 10 in the year 2024 (1 mark)

[5 marks]

(b) (i) $y = u\cos 3x + v\sin 3x$

$\Rightarrow \frac{dy}{dx} = -3u\sin 3x + 3v\cos 3x$ (2 marks)

$\Rightarrow \frac{d^2y}{dx^2} = -9u\cos 3x - 9v\sin 3x$ (2 marks)

so, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = -30\sin 3x$

$\Rightarrow -(6v - 12u)\sin 3x + (-6u + 12v)\cos 3x = -30\sin 3x$ (2 marks)

$\Rightarrow \quad 2u + v = 5$ and $u = 2v$ (2 marks)

$\Rightarrow u = 2$ and $v = 1$. (2 marks)

[10 marks]

(ii) the auxiliary equation of the different equation is

$$k^2 + 4k + 3 = 0$$ (1 mark)

$\Rightarrow \quad (k+3)(k+1) = 0$ (1 mark)

$\Rightarrow \quad k = -3 \text{ or } -1$

$\Rightarrow$ the complementary function is

$y = Ae^{-3x} + Be^{-x}$; A, B constants (2 marks)

General solution is $y = Ae^{-3x} + Be^{-x} + \sin 3x + 2\cos 3x.$ (1 mark)

[5 marks]

Total 25 marks

Specific Objectives: (a) 7, 8, 9, 10; (b) 1, 4, 5, 7

SECTION B

(MODULE 2)

Question 3

(a) (i) $$\frac{1}{(2r-1)(2r+1)} = \frac{A}{2r-1} + \frac{B}{2r+1}$$

$\Rightarrow \quad 1 = A(2r+1) + B(2r-1)$ (1 mark)

$\Rightarrow \quad 0 = 2A + 2B$ and $A - B = 1$ (2 marks)

$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{1}{2}$ (2 marks)

[5 marks]

(ii) $S = \sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \sum_{r=1}^{n} \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$ (1 mark)

$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots + \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

(3 marks)

$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$ (1 mark)

[5 marks]

(iii) As $n \to \infty$, $\frac{1}{2n+1} \to 0$ (2 marks)

Hence $S_\infty = \frac{1}{2}$ (1 mark)

[3 marks]

(b) (i) $S = 1(2) + 2(5) + 3(8) + \ldots$

In each term, 1st factor is in the natural sequence and the second factor differs by 3 (1 mark)

$\Rightarrow$ the nth term is $n(3n - 1)$ (1 mark)

[2 marks]

(ii) $S_n = \sum_{r=1}^{n} r(3r-1)$

for n = 1, $S_1 = \sum_{r=1}^{1} r(3r-1) = 1 x 2 = 2$

and $1^2 (1 + 1) = 1 x 2 = 2$ (1 mark)

hence $S_n = n^2 (n+1)$ is true for n = 1 (1 mark)

Assume $S_n = n^2 (n+1)$ for $n = k \in N$

that is, $S_k = k^2 (k+1)$ (1 mark)

Then, $S_{k+1} = \sum_{r=1}^{k+1} r(3r-1) = S_k + (k+1)(3k+2)$ (1 mark)

$= k^2(k+1) + (k+1)(3k+2)$ (1 mark)

$= (k+1)\left[k^2 + 3k + 2\right]$ (1 mark)

$\Rightarrow S_{k+1} = (k+1)\left[(k+1)(k+2)\right]$ (1 mark)

$= (k+1)^2 \left[(k+1)+1\right]$ (1 mark)

$\Rightarrow$ true for n = k+1 whenever it is assumed true for n = k, (1 mark)

$\Rightarrow$ true for all $n \in N$

$\Rightarrow S_n = n^2(n+1) \quad \forall n \in N$ (1 mark)

[10 marks]

Total 25 marks

Specific Objectives: (b) 1, 3, 5, 6, 10

Question 4

(a) (i) Let $S = \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$

S is a geometric series

$\Leftrightarrow \quad \frac{T_2}{T_1} = \frac{T_n}{T_{n-1}} = r$

i.e. S has common ratio r (1 mark)

$$\frac{\frac{1}{2^4}}{\frac{1}{2}} = \frac{\frac{1}{2^7}}{\frac{1}{2^4}}$$ (1 mark)

$$= \frac{1}{2^3}$$ (1 mark)

$\therefore$ S is geometric with common ratio $r = \frac{1}{2^3}$ (1 mark)

[4 marks]

(ii) $$S_n = \frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^{3n}\right]}{1-\left(\frac{1}{2}\right)^3}$$ (1 mark)

$$= \frac{\frac{1}{2}\left[1-\frac{1}{2^{3n}}\right]}{1-\frac{1}{8}}$$ (1 mark)

$$= \frac{1}{2} \times \frac{8}{7}\left[1-\frac{1}{2^{3n}}\right]$$ (1 mark)

$$= \frac{4}{7}\left[1-\frac{1}{2^{3n}}\right]$$ (1 mark)

[4 marks]

(b) (i) $f(x)=\cos 2x \quad \Rightarrow \quad f^{i}(x)=-2\sin 2x$ (1 mark)

$\Rightarrow \quad f^{ii}(x)=-4\cos 2x$ (1 mark)

$\Rightarrow \quad f^{iii}(x)=8\sin 2x$ (1 mark)

$\Rightarrow \quad f^{iv}(x)=16\cos 2x$ (1 mark)

so, $f(0)=1,\ f^{i}(0)=0,\ f^{ii}(x)=-4,\ f^{iii}(0)=0,\ f^{iv}(0)=16$ (1 mark)

Hence, by Maclaurin's Theorem,

$$\cos 2x = 1-\frac{4x^2}{2!}+\frac{16x^4}{4!}-$$ (1 mark)

$$= 1-2x^2+\frac{2}{3}x^4$$ (1 mark)

[7 marks]

(c) (i) $\sqrt{\left(\frac{1+x}{1-x}\right)}$

$= (1+x)^{½}\ (1-x)^{-½}$ (1 mark)

$$= \left(1+\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3\ldots\right)\left(1+\frac{1}{2}x-\frac{3}{8}x^2+\frac{5}{16}x^3\ldots\right)$$ (2 marks)

$$= 1+x+\frac{1}{2}x^2+\frac{1}{2}x^3$$ (2 marks)

for $-1<x<1$ (1 mark)

[6 marks]

(ii) $\sqrt{\frac{1.02}{0.98}}=\sqrt{\frac{102}{98}}=\frac{1}{7}\sqrt{51}$ (1 mark)

$\sqrt{51} = 7\sqrt{\frac{1+x}{1-x}}$ where $x=0.02$ (1 mark)

$\Rightarrow \sqrt{51} = 7\left\{1+0.02+\frac{1}{2}(0.02)^2+\frac{1}{2}(0.02)^3\right\}$ (1 mark)

$= 7.14141$ (5 d.p.) (1 mark)

[4 marks]

Specific Objectives: (b) 5, 9, 11; (c)3, 4

Total 25 marks

SECTION C

(MODULE 3)

Question 5

(a) (i) P (First card drawn has even number) $= \frac{5}{10} = \frac{1}{2}$ (1 mark)

P (Second card drawn has even number) $= \frac{4}{9}$ (2 marks)

$\therefore$ P (Both cards have even numbers) $= \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)$

$= \frac{2}{9}$ (1 mark)

[4 marks]

(ii) P (Both cards have odd numbers) $= \frac{2}{9}$ (1 mark)

P (One card has odd and the other has even i.e. both cards do not have odd or do not have even numbers) $= 1 - 2\left(\frac{2}{9}\right)$ (2 marks)

$= \frac{5}{9}$ (1 mark)

[4 marks]

(b) (i) a) $\frac{82}{150} = 0.547$ [2 marks]

b) $\frac{39}{150} + \frac{75}{150} = 0.76$ [4 marks]

c) $\frac{82}{150} + \frac{39}{150} - \frac{27}{150} = 0.267$ [5 marks]

d) $\frac{27}{82} = 0.329$ [2 marks]

(ii) $\frac{82}{150} \times \frac{81}{149} = 0.297$ [4 marks]

Total 25 marks

Specific Objectives: (a) , 5, 6, 7, 8, 9, 10, 11, 12, 13

Question 6

(a) $$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$$

$$5\left(2x-2\right)-x\left(x^2+2x+3\right)+3\left(2x+4+6\right) = 0 \qquad \text{(5 marks)}$$

$$x^3+2x^2-13x-20 = 0 \qquad \text{(1 mark)}$$

Subs $x = -4$, $(-4)^3 + 2(-4)^2 - 13(-4) - 20 = 0$

$$\left(x+4\right)\left(x^2-2x-5\right) = 0 \qquad \text{(2 marks)}$$

$$x = -4$$

$$x = \frac{2 \pm \sqrt{24}}{2}$$

$$x = 1 \pm \sqrt{6} \qquad \text{(2 marks)}$$

[10 marks]

(b) Writing the equations in matrix form.

$$\begin{pmatrix} 1 & -4 & -2 \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ 3 \\ -2 \end{pmatrix}$$

The augmented matrix is

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & -1 & -2 \end{array}\right)$$

(1 mark)

Eliminate x_1 from Row 2 and Row 3
Subtract 2 x Row_1 from Row_2 and 3 x Row_1 from Row_3 to give

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 0 & 9 & 6 & -39 \\ 0 & 14 & 5 & -65 \end{array}\right) \qquad \text{(2 marks)}$$

Row 3 $-\dfrac{14}{9}$ Row 2 gives

$$\left(\begin{array}{ccc|c} 1 & -4 & -2 & 21 \\ 0 & 9 & 6 & -39 \\ 0 & 0 & \frac{-13}{3} & \frac{-13}{3} \end{array}\right)$$

(2 marks)

Referencing the matrix gives

$$\begin{pmatrix} 1 & -4 & -2 \\ 0 & 9 & 6 \\ 0 & 0 & \frac{-13}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 21 \\ -39 \\ \frac{-13}{3} \end{pmatrix}$$

$x_1 = 3$, $x_2 = -5$, $x_3 = 1$
by "back substitution".

$\left[\begin{array}{ll} x_3 & -\text{ 1 mark} \\ x_2,\ x_1 & -\text{ 2 marks each} \end{array}\right]$

(5 marks)
[10 marks]

(c) (i) $\dfrac{4-2i}{1-3i} = \dfrac{(4-2i)(1+3i)}{(1-3i)(1+3i)}$ (1 mark)

$= \dfrac{4+12i-2i-6i^2}{1-9i^2}$ (1 mark)

$= \dfrac{4+10i+6}{1+9}$ (1 mark)

$= \dfrac{10+10i}{10}$

$= 1 + i$ (1 mark)

[4 marks]

(ii) arg is $\tan^{-1}(\frac{1}{1}) = \frac{\pi}{4}$ (1 mark)

[1 mark]

Total 25 marks

Specific Objectives: (b) 1, 2, 3, 4, 5, 6, 7, 8; (c) 4, 5, 6, 7

02234032/CAPE/MS/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03B

SOLUTIONS

&

MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) The volume of the air freshener decreases exponentially with time.

At $t = 0$, the volume is A cm^3. 1 mark
As $t \to \infty$, the volume $\to 0$. 1 mark
[2 marks]

(b) $\int \frac{1}{V}\, dV = \int -k dt$ 1 mark

$\ln V = -kt + c$ 2 marks

$V = e^{-kt+c}$ 1 mark
$= e^{-kt} e^{c}$
$= Ae^{-kt}$ where $A = e^{c}$. 1 mark
[5 marks]
OR

i.e. $V = Ae^{-kt}$, as required

OR

$\ln V = -kt + \ln A$ 1 mark
$\ln V - \ln A = -kt$ 1 mark
$\ln \frac{V}{A} = -kt$ 1 mark
$\frac{V}{A} = e^{-kt}$ 1 mark

$V = Ae^{-kt}$, as required 1 mark
[5 marks]

(c) $V = Ae^{-kt}$
At $t = 0$, $V = 64$:
$64 = A(1)$ 1 mark
$A = 64$
$\therefore V = 64e^{-kt}$ 1 mark

At $t = 6$, $V = 32$ 1 mark
$32 = 64e^{-6k}$

$\frac{1}{2} = e^{-6k}$ 1 mark

$\ln \frac{1}{2} = -6k$ 1 mark

$k = -\frac{1}{6} \ln \frac{1}{2}$ 1 mark

$\therefore \; V = 64e^{-\left(-\frac{1}{6} \ln \frac{1}{2}\right) t}$ 1 mark

$V = 64e^{\left(\ln \frac{1}{2}\right)\left(\frac{t}{6}\right)}$ 1 mark

[8 marks]

(d) Now $V = 64e^{\left(\ln \frac{1}{2}\right)\left(\frac{t}{6}\right)}$

$\Rightarrow \quad V = 64\left(\frac{1}{2}\right)^{\frac{t}{6}}$ 1 mark

Block is ineffective for V = 6 1 mark

$\therefore \quad 6 = 64\left(\frac{1}{2}\right)^{\frac{t}{6}}$ 1 mark

$\frac{3}{32} = \left(\frac{1}{2}\right)^{\frac{t}{6}}$

$\ln \frac{3}{32} = \frac{t}{6} \ln \left(\frac{1}{2}\right)$

$t = \dfrac{6 \ln \frac{3}{32}}{\ln \frac{1}{2}}$ 1 mark

≈ 20.49

≈ 20

Ans 20 weeks 1 mark

[5 marks]

Total 20 marks

Specific Objectives: (a) 3, 7, 8, 9, 11; (c) 11

SECTION B

(MODULE 2)

Question 2

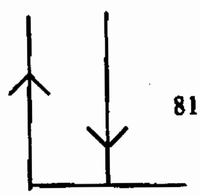
(a) $S_\infty = \dfrac{a}{1-r}$

So $\dfrac{a}{1-r} = 10a$ — 1 mark

$\Rightarrow 10(1-r) = 1$ — 1 mark

$\Rightarrow r = 9/10$ — 1 mark

[3 marks]



(b) (i) a) $h_1 = 2 \text{ x } 81 \text{ cm} = 162 \text{ cm}$ — 1 mark

b) $h_2 = \dfrac{2}{3} \text{ x } 81 \text{ x } 2 \text{ cm} = \dfrac{2}{3} h_1$ — 1 mark

c) $h_3 = \dfrac{2}{3} \text{ x } \dfrac{2}{3} \text{ x } 81 \text{ x } 2 \text{ cm} = \dfrac{2}{3} h_2$ — 1 mark

[3 marks]

(ii) a) Hence

$h_n = \dfrac{2}{3} h_{n-1}$ — 1 mark

b) $h_{n-1} = \dfrac{2}{3} h_{n-2}$ — 1 mark

[2 marks]

(iii) So, $h_n = \dfrac{2}{3} h_{n-1}, n > 1$

$= \dfrac{2}{3} \text{ x } \dfrac{2}{3} h_{n-2}, n > 2$ — 1 mark

$= \underbrace{\dfrac{2}{3} \text{ x } \dfrac{2}{3} \text{ x } \ldots \text{ x } \dfrac{2}{3}}_{n-1 \text{ times}} h_1$ — 1 mark

$= \left(\dfrac{2}{3}\right)^{n-1} h_1$ — 1 mark

[3 marks]

(iv) Distance $= h_1 + h_2 + \ldots + h_n$ 1 mark

$$= h_1 + \frac{2}{3}h_1 + \left(\frac{2}{3}\right)^2 h_1 + \ldots + \left(\frac{2}{3}\right)^{n-1} h_1$$ 1 mark

$$= h_1\left(1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots + \left(\frac{2}{3}\right)^{n-1}\right)$$ 1 mark

$$= h_1\left(\frac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}}\right)$$ 1 mark

$$= 162\left(\frac{1 - \left(\frac{2}{3}\right)^n}{\frac{1}{3}}\right)$$ 1 mark

$$= 486\left(1 - \left(\frac{2}{3}\right)^n\right)\text{cm}$$ 1 mark

[6 marks]

(v) As $n \to \infty$, $\left(\frac{2}{3}\right)^n \to 0$. 2 marks

Hence, when the ball stops bouncing the distance is approximately 486 cm. 1 mark

[3 marks]

Total 20 marks

Specific Objectives: (a)3, 4; (b) 5, 9, 10, 12; GO:6

SECTION C

(MODULE 3)

Question 3

(a) (i)

Number of ways of choosing 1^{st} song is	6	
Number of ways of choosing 2^{nd} song is	5	
Number of ways of choosing 3^{rd} song is	4	
Number of ways of choosing 4^{th} song is	3	
Number of ways of choosing 5^{th} song is	2	
Number of ways of choosing 6^{th} song is	1	
		1 mark

Total number of ways is $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$ 1 mark

[2 marks]

(ii)

Number of ways of choosing 1^{st} song is	13	1 mark
Number of ways of choosing 2^{nd} song is	12	
Number of ways of choosing 3^{rd} song is	11	
Number of ways of choosing 4^{th} song is	10	
Number of ways of choosing 5^{th} song is	9	
Number of ways of choosing 6^{th} song is	8	1 mark

Total number of ways is $13 \times 12 \times 11 \times 10 \times 9 \times 8 = {}^{13}P_7$ 1 mark

$= 1\ 235\ 520$

[3 marks]

(b) (i)

P (None correct) = P (r = 0) 1 mark

P (incorrect guess) = 1 – 0.5 1 mark

$$P(r=0) = \frac{4!}{(0!)\,(4-0)!}\,(0.50)^0\,(1-0.50)^{4-0}$$ 1 mark

$$= \frac{4 \times 3 \times 2 \times 1}{(1)(4 \times 3 \times 2 \times 1)}\,(1)(0.50)^4$$

$$= (0.50)^4$$

$$= 0.0625$$ 1 mark

$$P(r \geq 1) = 1 - 0.0625$$
$$= 0.938$$ 1 mark

[5 marks]

(ii) $P(r=1) = \dfrac{4!}{(0!)\,(4-1)!}\,(0.50)^1\,(1-0.50)^{4-1}$ 1 mark

$= \dfrac{4 \times 3 \times 2 \times 1}{(1)\,(3 \times 2 \times 1)}\,(0.50)\,(0.50)^3$

$= 4\,(0.50)^4 = 0.2500$ 1 mark

[2 marks]

(c) Augmented matrix $= \left(\begin{array}{ccc|c} 1 & 4 & 1 & 7 \\ 1 & -1 & 2 & 9 \\ 2 & 1 & -1 & 1 \end{array}\right)$ 1 mark

$r_3 - 2r_1 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 1 & -1 & 2 & 9 \\ 0 & -1 & -3 & -13 \end{array}\right)$ 1 mark

$r_2 - r_1 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 2 & 2 \\ 0 & -1 & -3 & -13 \end{array}\right)$ 1 mark

$2r_3 - r_2 \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -2 & 1 & 2 \\ 0 & 0 & -7 & -28 \end{array}\right)$ 1 mark

Re-writing the matrix gives

$$\begin{aligned} x + y + z &= 7 \\ -2y + z &= 2 \\ -7z &= -28 \end{aligned}$$

Solving for x, y and z gives $z = 4$, $y = 1$, $x = 2$ 3 marks

$P = (2, 1, 4)$ 1 mark

[8 marks]

Total 20 marks

Specific Objectives: (a) 2, 4, 7, 9, 10 (b) 3, 4, 5; GO: 5

FORM TP 2007249

TEST CODE **02234010**

MAY/JUNE 2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 01

2 hours

21 MAY 2007 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

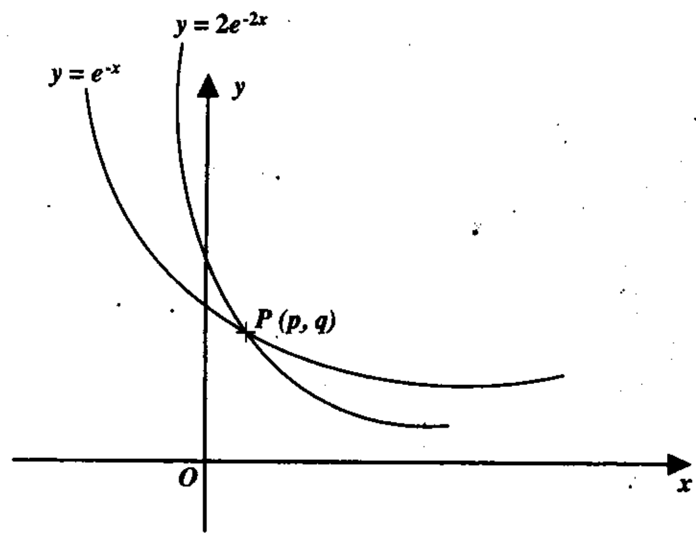
Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. (a) In the diagram below, **not drawn to scale**, the curves $y = 2e^{-2x}$ and $y = e^{-x}$ intersect at $P\ (p, q)$.



Determine the values of p and q. $(\ln 2, \frac{1}{2})$ **[5 marks]**

(b) Solve $2^{2x+1} = 6$ for $x \in \mathbf{R}$. $x = \frac{\ln 3 - 1}{2}$ **[3 marks]**

Total 8 marks

2. (a) The parametric equations of a curve are $x = \frac{4}{t}$ and $y = 2t^2 + 2$.

Find the gradient of the curve at the point (4, 4). $t = 1$ **[5 marks]**

(b) Differentiate with respect to x $\frac{dy}{dx}$ $= -1$

$y = \tan^2 3x + \ln (x^3)$. **[4 marks]**

$2\tan 3x\,(\sec^2 3x) + \frac{3}{x}$ **Total 9 marks**

3. (a) Express $\frac{5}{(3+x)(2-x)}$ in the form $\frac{P}{3+x} + \frac{Q}{2-x}$, where P and Q are constants. $P = 1$, $Q = 1$ **[3 marks]**

$\frac{1}{3+x} + \frac{1}{2-x}$

(b) Hence, find $\int \frac{5}{(3+x)(2-x)}\,dx$. **[3 marks]**

Total 6 marks

$\ln|3+x| - \ln|2-x| + C$

GO ON TO THE NEXT PAGE

$\frac{(2x-5)^5}{?2}(2x+1)$

4. Obtain the following:

(a) $\int x\,(2x-5)^4\,dx$, **by substituting $u = 2x - 5$.** **[5 marks]**

(b) $\int x \sec^2 x\,dx$, **using integration by parts.** **[4 marks]**

$x \tan x + \ln|\cos x| + K$

Total 9 marks

5. The cost \$$c$ of manufacturing x items may be modelled by the differential equation

$\frac{dc}{dx} + 2c = 10x.$

$c = 5x + 100$

By using a suitable integrating factor, solve the differential equation, given that there is a cost of \$100 when no items are produced.

Total 8 marks

Section B (Module 2)

Answer ALL questions.

6. A sequence $\{u_n\}$ is defined by $u_{n+1}\,u_n = 2^n$, $n \geq 1$.

(a) Prove that $u_{n+2} = 2u_n$. **[4 marks]**

(b) If $u_1 = 2$, find u_3 and u_5. **[4 marks]**

Total 8 marks

7. The sum of the first and third terms of a GP is 50, and the sum of the second and fourth terms is 150. For this GP, find

(a) the common ratio **[4 marks]**

(b) the first term **[2 marks]**

(c) the sum of the first five terms. **[2 marks]**

Total 8 marks

8. Find the term independent of x in the expansion of $\left(2x^2 - \frac{5}{x^3}\right)^{10}$. **[7 marks]**

Total 7 marks

9. (a) Use the fact that ${}^nC_k = \frac{n!}{k!(n-k)!}$ to express, in terms of factorials,

(i) the coefficient u of x^n in the expansion of $(1 + x)^{2n}$. **[2 marks]**

(ii) the coefficient v of x^n in the expansion of $(1 + x)^{2n-1}$. **[3 marks]**

(b) Hence, show that $u = 2v$. **[4 marks]**

Total 9 marks

10. (a) Given that $f(r) = \frac{2r+1}{(r-1)(r-2)}$, prove that $f(r) - f(r+1) = \frac{2(r+3)}{r(r-1)(r-2)}$.

[4 marks]

(b) Hence, find $\sum_{r=3}^{n} \frac{r+3}{r(r-1)(r-2)}$. **[4 marks]**

Total 8 marks

Section C (Module 3)

Answer ALL questions.

11. (a) Determine the number of ways in which the letters of the word STATISTICS may be arranged so that the vowels are placed together. [3 marks]

(b) A team of five is chosen at random from 4 boys and 6 girls. Calculate the number of ways that this team can be chosen to include **at least** 3 girls. [5 marks]

Total 8 marks

12. Two unbiased dice each with six faces are tossed randomly one after the other.

(a) Determine the set of possible outcomes. [2 marks]

(b) Find the probability that

(i) the product of the numbers on the two dice is a multiple of 5 [2 marks]

(ii) the second die shows the number 2 [1 mark]

(iii) the product of the numbers on the two dice is a multiple of 5 **OR** the second die shows the number 2. [2 marks]

Total 7 marks

13. The matrices X and Y are given by $X = \begin{pmatrix} 4 & 2 & 1 \\ -5 & 6 & 8 \\ 7 & 9 & -3 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 6 \\ 5 & 2 \\ 4 & 6 \end{pmatrix}$.

Calculate

(a) the determinant of X [4 marks]

(b) $Y^T X$. [3 marks]

Total 7 marks

14. Y and X are 3 x 1 matrices and are related by the equation $Y = AX$, where $A = \begin{pmatrix} 1 & 0 & 3 \\ 7 & 5 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ is a non-singular matrix.

Find

(a) A^{-1} **[6 marks]**

(b) X, when $Y = \begin{pmatrix} 10 \\ 12 \\ 8 \end{pmatrix}$. **[3 marks]**

Total 9 marks

15. Air is pumped into a spherical balloon of radius r cm at the rate of 275 cm^3 per second. When $r = 10$, calculate the rate of increase of

(a) the radius r **[5 marks]**

(b) the surface area S. **[4 marks]**

[The volume V and surface area S of a sphere of radius r are given by $V = \dfrac{4\pi r^3}{3}$ and $S = 4\pi r^2$ respectively.]

Total 9 marks

END OF TEST

TEST CODE **02234020**

FORM TP 2007250

MAY/JUNE 2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

2 hours

30 MAY 2007 (p.m)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 5 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) Solve, for $x > 0$, the equation $3 \log_8 x = 2 \log_x 8 - 5$. **[8 marks]**

(b) (i) Copy and complete the table below for values 2^x and e^{-x} using a calculator, where necessary. **Approximate all values to 2 decimal places.**

x	**–1.0**	**0**	**0.5**	**1.0**	**1.5**	**2.0**	**2.5**	**3.0**
2^x		1.00	1.41	2.00		4.00		8.00
e^{-x}	2.72			0.37	0.22			0.05

[3 marks]

(ii) On the same pair of axes and using a scale of 4 cm for 1 unit on the x-axis, 4 cm for 1 unit on the y-axis, draw the graphs of the two curves $y = 2^x$ and $y = e^{-x}$ for $-1 \leq x \leq 3, x \in \mathbf{R}$. **[5 marks]**

(iii) Use your graphs to find

a) the value of x satisfying $2^x - e^{-x} = 0$ **[2 marks]**

b) the range of values of x for which $2^x - e^{-x} < 0$. **[2 marks]**

Total 20 marks

2. (a) Show that for $n \geq 2$, $\tan^n x = \tan^{n-2} x \sec^2 x - \tan^{n-2} x$. **[3 marks]**

(b) Find $\frac{dy}{dx}$ when $y = \tan^n x$. **[3 marks]**

(c) Let $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx, n \geq 2$.

(i) By using the result in (a) above, show that $I_n + I_{n-2} = \frac{1}{n-1}$. **[7 marks]**

(ii) Hence evaluate I_4. **[7 marks]**

Total 20 marks

Section B (Module 2)

Answer BOTH questions.

3. (a) The sequence $\{u_n\}$ is given by $u_1 = 1$ and $u_{n+1} = (n+1)\,u_n$, $n \geq 1$.

Prove by Mathematical Induction that $u_n = n!\ \ \forall n \in \mathbf{N}$. **[9 marks]**

(b) Given that the sum of the first n terms of a series, S, is $9 - 3^{2-n}$,

(i) find the n-th term of S **[5 marks]**

(ii) show that S is a geometric progression **[2 marks]**

(iii) find the first term and common ratio of S **[2 marks]**

(iv) deduce the sum to infinity of S. **[2 marks]**

Total 20 marks

4. (a) The function f is given by $f: x \rightarrow x^4 - 4x + 1$. Show that

(i) $f(x) = 0$ has a root α in the interval $(0, 1)$ **[4 marks]**

(ii) if x_1 is a first approximation to α of $f(x) = 0$ in $(0, 1)$, the Newton-Raphson method gives a second approximation x_2 in $(0, 1)$ satisfying $x_2 = \dfrac{3x_1^{\,4} - 1}{4\left(x_1^{\,3} - 1\right)}$. **[5 marks]**

(b) John's father gave him a loan of \$10 800 to buy a car. The loan was to be repaid by 12 unequal monthly instalments, starting with an initial payment of \$$P$ in the first month. There is no interest charged on the loan, but the instalments increase by \$60 per month.

(i) Show that $P = 570$. **[5 marks]**

(ii) Find, in terms of n, $1 \leq n \leq 12$, an expression for the remaining debt on the loan after John has paid the n-th instalment. **[6 marks]**

Total 20 marks

Section C (Module 3)

Answer BOTH questions.

5. (a) A bag contains 5 white marbles and 5 black marbles. Six marbles are chosen at random.

(i) Determine the number of ways of selecting the six marbles if there are no restrictions. **[2 marks]**

(ii) Find the probability that the marbles chosen contain more black marbles than white marbles. **[4 marks]**

(b) The table below summarises the programme preference of 100 television viewers.

Television Preference	Number of Males	Number of Females	Total
Matlock	20	10	30
News	14	18	32
Friends	18	20	38
Total	52	48	100

Determine the probability that a person selected at random

(i) is a female **[2 marks]**

(ii) is a male or likes watching the News **[4 marks]**

(iii) is a female that likes watching Friends **[2 marks]**

(iv) does not like watching Matlock. **[2 marks]**

(c) The table below lists the probability distribution of the number of accidents per week on a particular highway.

Number of Accidents Per Week	0	1	2	3	4	5
Probability	0.25	0	0.10	p	0.30	0.15

(i) Calculate the value of p. **[2 marks]**

(ii) Determine the probability that there are more than 3 accidents in a week. **[2 marks]**

Total 20 marks

6. (a) A system of equations is given by

$$x + y + z = 10$$
$$3x - 2y + 3z = 35$$
$$2x + y + 2z = \alpha$$

where α is a real number.

(i) Write the system in matrix form. **[1 mark]**

(ii) Write down the augmented matrix. **[1 mark]**

(iii) Reduce the augmented matrix to echelon form. **[3 marks]**

(iv) Deduce the value of α for which the system is consistent. **[1 mark]**

(v) Find **ALL** solutions corresponding to this value of α. **[4 marks]**

(b) Given $A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$,

find

(i) $kI - A$, where I is the 3 x 3 Identity matrix and k is a real number **[3 marks]**

(ii) the values of k for which $|\, kI - A \,| = 0$.

[7 marks]

Total 20 marks

END OF TEST

TEST CODE **02234020**

FORM TP 2008243

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

28 MAY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Differentiate with respect to x

(i) $e^{4x} \cos \pi x$ **[4 marks]**

(ii) $\ln \dfrac{x^2 + 1}{\sqrt{x}}$. **[4 marks]**

(b) Given $y = 3^{-x}$, show, by using logarithms, that

$$\frac{dy}{dx} = -3^{-x} \ln 3.$$ **[5 marks]**

(c) (i) Express in partial fractions

$$\frac{2x^2 - 3x + 4}{(x-1)(x^2+1)}.$$ **[7 marks]**

(ii) Hence, find

$$\int \frac{2x^2 - 3x + 4}{(x-1)(x^2+1)} \, dx.$$ **[5 marks]**

Total 25 marks

2. (a) Solve the differential equation

$$\frac{dy}{dx} + y = e^{2x}.$$ **[5 marks]**

(b) The gradient at the point (x, y) on a curve is given by

$$\frac{dy}{dx} = e^{4x}.$$

Given that the curve passes through the point (0, 1), find its equation. **[5 marks]**

(c) Evaluate $\int_1^e x^2 \ln x \, dx$, writing your answer in terms of e. **[7 marks]**

GO ON TO THE NEXT PAGE

(d) (i) Use the substitution $v = 1 - u$ to find

$$\int \frac{du}{\sqrt{1 - u}}.$$ **[3 marks]**

(ii) Hence, or otherwise, use the substitution $u = \sin x$ to evaluate

$$\int_0^{\pi/2} \sqrt{1 + \sin x} \quad dx.$$ **[5 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) A sequence $\{u_n\}$ is defined by the recurrence relation

$$u_{n+1} = u_n + n, \; u_1 = 3, \quad n \in \mathrm{N}.$$

(i) State the first FOUR terms of the sequence. **[3 marks]**

(ii) Prove by mathematical induction, or otherwise, that

$$u_n = \frac{n^2 - n + 6}{2}.$$ **[8 marks]**

(b) A GP with first term a and common ratio r has sum to infinity 81 and the sum of the first four terms is 65. Find the values of a and r. **[6 marks]**

(c) (i) Write down the first FIVE terms in the power series expansion of $\ln (1 + x)$, stating the range of values of x for which the series is valid. **[3 marks]**

(ii) a) Using the result from (c) (i) above, obtain a similar expansion for $\ln (1 - x)$. **[2 marks]**

b) Hence, prove that

$$\ln \left(\frac{1+x}{1-x}\right) = 2 \left(x + \frac{1}{3} x^3 + \frac{1}{5} x^5 + \ldots\right).$$ **[3 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) (i) Show that the function $f(x) = x^3 - 3x + 1$ has a root α in the closed interval [1, 2]. **[3 marks]**

(ii) Use the Newton-Raphson method to show that if x_1 is a first approximation to α in the interval [1, 2], then a second approximation to α in the interval [1, 2] is given by

$$x_2 = \frac{2x_1^3 - 1}{3x_1^2 - 3}.$$ **[5 marks]**

(b) (i) Use the binomial theorem or Maclaurin's theorem to expand $(1+x)^{-\frac{1}{2}}$ in ascending powers of x **as far as the term in x^3**, stating the values of x for which the expansion is valid. **[4 marks]**

(ii) Obtain a similar expansion for $(1-x)^{\frac{1}{2}}$. **[4 marks]**

(iii) Prove that if x is so small that x^3 and higher powers of x can be neglected, then

$$\sqrt{\frac{1-x}{1+x}} \approx 1 - x + \frac{1}{2}x^2.$$ **[5 marks]**

(iv) Hence, by taking $x = \frac{1}{17}$, show, **without using calculators or tables**, that $\sqrt{2}$ is approximately equal to $\frac{1635}{1156}$. **[4 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) A cricket selection committee of 4 members is to be chosen from 5 former batsmen and 3 former bowlers.

In how many ways can this committee be selected so that the committee includes AT LEAST

(i) ONE former batsman? **[8 marks]**

(ii) ONE batsman and ONE bowler? **[3 marks]**

(b) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 3 & -1 \end{pmatrix} \text{ and } \mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & -3 \\ -1 & 3 & -1 \end{pmatrix},$$

(i) determine EACH of the following matrices:

a) $\mathbf{A} - \mathbf{B}$ **[2 marks]**

b) $\mathbf{AM}$ **[3 marks]**

(ii) deduce from (i) b) above the inverse $\mathbf{A}^{-1}$ of the matrix **A** **[3 marks]**

(iii) find the matrix **X** such that $\mathbf{AX} + \mathbf{B} = \mathbf{A}$. **[6 marks]**

Total 25 marks

6. (a) (i) Express the complex number

$$\frac{2-3i}{5-i} \text{ in the form } \lambda\,(1-i).$$ **[4 marks]**

(ii) State the value of λ. **[1 mark]**

(iii) Verify that $\left(\frac{2-3i}{5-i}\right)^4$ is a real number and state its value. **[5 marks]**

(b) The complex number z is represented by the point T in an Argand diagram.

Given that $z = \frac{1}{3+it}$ where t is a variable and $\bar{z}$ denotes the complex conjugate of z, show that

(i) $z + \bar{z} = 6\,z\bar{z}$ **[7 marks]**

(ii) as t varies, T lies on a circle, and state the coordinates of the centre of this circle. **[8 marks]**

Total 25 marks

END OF TEST

FORM TP 2008244

TEST CODE **02234032**

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

19 MAY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) The parametric equations of a curve are given by $x = 3t^2$ and $y = 6t$.

   (i) Find the value of $\frac{dy}{dx}$ at the point P on the curve where $y = 18$. **[5 marks]**

   (ii) Find the equation of the normal to the curve at P. **[3 marks]**

   (b) In an experiment it was discovered that the volume, V cm³, of a certain substance in a room after t seconds may be determined by the equation

   $$V = 60\, e^{0.04t}.$$

   (i) Find $\frac{dV}{dt}$ in terms of t. **[3 marks]**

   (ii) Determine the rate at which the volume

   a) increases after 10 seconds **[1 mark]**

   b) is increasing when it is 180 cm³. **[3 marks]**

   (iii) Sketch the graph of $V = 60\, e^{0.04t}$ showing the point(s) of intersection, where they exist, with the axes. **[5 marks]**

Total 20 marks

GO ON TO THE NEXT PAGE

SECTION B (Module 2)

Answer this question.

2. (a) Matthew started a savings account at a local bank by depositing \$5 in the first week. In each succeeding week after the first, he added twice the amount deposited in the previous week.

(i) Derive an expression for

a) the amount deposited in the r^{th} week, **in terms of r** **[3 marks]**

b) the TOTAL amount in the account after n weeks, **in terms of n.** **[3 marks]**

(ii) Calculate the MINIMUM number, n, of weeks it would take for the amount in the account to **exceed** \$1000.00 if **no** withdrawal is made. **[3 marks]**

(b) The series S is given by

$$S = 1\frac{1}{2} + 3\frac{1}{4} + 5\frac{1}{8} + 7\frac{1}{16} + \ldots$$

(i) Express S as the sum of an AP and a GP. **[3 marks]**

(ii) Find the sum of the first n terms of S. **[3 marks]**

(c) (i) Use the binomial theorem to expand $\frac{1}{1-y}$ as a power series in y **as far as the term in y^4.** **[2 marks]**

(ii) Given that the Maclaurin series expansion for $\cos x$ is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

find the first THREE non-zero terms in the power series expansion of $\sec x$. **[3 marks]**

Total 20 marks

GO ON TO THE NEXT PAGE

SECTION C (Module 3)

Answer this question.

3. (a) (i) By considering the augmented matrix for the following system of equations, determine the value of k for which the system is consistent.

$$\begin{aligned} x + 3y + 5z &= 2 \\ x + 4y - z &= 1 \\ y - 6z &= k \end{aligned}$$

[5 marks]

(ii) Find ALL the solutions to the system for the value of k obtained in (i) above. **[4 marks]**

(b) The probability that a person selected at random

– owns a car is 0.25
– is self-employed is 0.40
– is self-employed OR owns a car is 0.6.

(i) Determine the probability that a person selected at random owns a car AND is self-employed. **[4 marks]**

(ii) Stating a reason in EACH case, determine whether the events 'owns a car' and 'is self-employed' are

a) independent events **[4 marks]**

b) mutually exclusive events. **[3 marks]**

Total 20 marks

END OF TEST

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FORM TP 2008243

TEST CODE **22234020**

MAY/JUNE 2008

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

15 JULY 2008 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Write your solutions, with full working, in the answer booklet provided.

4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Find the **exact** values of x such that

$$e^x + 7e^{-x} = 8.$$ **[5 marks]**

(b) Given that $u = e^{2x} + e^{-2x}$, $v = e^{2x} - e^{-2x}$, show that

$$\frac{d^2u}{dx^2} + \frac{d^2v}{dx^2} = 2\left(\frac{du}{dx} + \frac{dv}{dx}\right).$$ **[7 marks]**

(c) (i) Differentiate with respect to x

$$(x \ln x) \sin^{-1} 2x.$$ **[4 marks]**

(ii) A curve **C** has parametric equations

$$x = 3t^2 + 5, \quad y = 2t^3 + 6t.$$

a) Show that $\frac{dy}{dx} = t + \frac{1}{t}$. **[3 marks]**

b) Show that **C** has points of inflexion at (8, 8) and (8, –8). **[6 marks]**

Total 25 marks

2. (a) (i) Find $\int \frac{1}{x} \ln x \, dx$. **[3 marks]**

(ii) Solve the differential equation

$$x^2 \frac{dy}{dx} + xy = \ln x.$$ **[5 marks]**

(b) (i) Find the values of the constants m and n, given that $y = m \cos x + n \sin x$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 10 \sin x.$$ **[5 marks]**

(ii) Hence, find the general solution of the differential equation. **[3 marks]**

(c) (i) Express $\dfrac{2 + 3x - x^2}{(x - 1)(x^2 + 1)}$ in partial fractions. **[6 marks]**

(ii) Hence, find $\displaystyle\int \frac{2 + 3x - x^2}{(x - 1)(x^2 + 1)}\, dx$. **[3 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) (i) Let $\mathbf{S} = \sum_{r=1}^{n} r$ and $\mathbf{T} = \sum_{r=1}^{n} (n + 1 - r)$.

a) Show that $\mathbf{T} = \mathbf{S}$. **[2 marks]**

b) Deduce that $\mathbf{S} = \dfrac{1}{2} n(n + 1)$. **[3 marks]**

(ii) Use the principle of mathematical induction to prove that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n + 1)(2n + 1).$$ **[7 marks]**

(iii) Hence, prove that $\sum_{r=1}^{n} 2r\,(3r + 1) = 2n(n + 1)^2$. **[4 marks]**

(b) (i) Show that the equation $x^3 + 3x^2 + 6x - 3 = 0$ has a root α between 0 and 1. **[2 marks]**

(ii) Prove that α is the only real root. **[3 marks]**

(iii) Using TWO iterations of the Newton-Raphson method, find α correct to 2 decimal places. **[4 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) The sequence $\{a_n\}$ of **positive numbers** is defined by

$$a_{n+1} = \frac{4(1+a_n)}{4+a_n}, \; a_1 = \frac{3}{2}.$$

(i) Find a_2 and a_3. **[2 marks]**

(ii) Express $a_{n+1} - 2$ in terms of a_n. **[2 marks]**

(iii) Given that $a_n < 2$ for all n, show that

a) $a_{n+1} < 2$ **[3 marks]**

b) $a_n < a_{n+1}$. **[6 marks]**

(b) Find the term independent of x in the binomial expansion of $(x^2 - \frac{6}{x^3})^{15}$.

[You may leave your answer in the form of factorials and powers, for example, $\frac{15!}{2!} \times 8^5$.]

[6 marks]

(c) Use the binomial theorem to find the difference between 2^{10} and $(2.002)^{10}$ correct to 5 decimal places. **[6 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Four-digit numbers are formed from the digits 1, 2, 3, 4, 7, 9.

(i) How many 4-digit numbers can be formed if

a) the digits, 1, 2, 3, 4, 7, 9, can all be repeated? **[2 marks]**

b) none of the digits, 1, 2, 3, 4, 7, 9, can be repeated? **[2 marks]**

(ii) Calculate the probability that a 4-digit number in (a) (i) b) above is even. **[3 marks]**

(b) A father and son practise shooting at basketball, and score when the ball hits the basket. The son scores 75% of the time and the father scores 4 out of 7 tries. If EACH takes one shot at the basket, calculate the probability that only ONE of them scores. **[6 marks]**

GO ON TO THE NEXT PAGE

(c) (i) Find the values of $h, k \in \mathbf{R}$ such that $3 + 4i$ is a root of the quadratic equation

$$z^2 + hz + k = 0.$$ **[6 marks]**

(ii) Use De Moivre's theorem for $(\cos \theta + i \sin \theta)^3$ to show that

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta.$$ **[6 marks]**

Total 25 marks

6. (a) Solve for x the equation

$$\begin{vmatrix} 1 & 1 & 1 \\ x & 2 & 1 \\ x^3 & 8 & 1 \end{vmatrix} = 0.$$ **[12 marks]**

(b) The Popular Taxi Service in a certain city provides transportation for tours of the city using cars, coaches and buses. Selection of vehicles for tours of distances (in km) is as follows:

x cars, $2y$ coaches and $3z$ buses cover 34 km tours.
$2x$ cars, $3y$ coaches and $4z$ buses cover 49 km tours.
$3x$ cars, $4y$ coaches and $6z$ buses cover 71 km tours.

(i) Express the information above as a matrix equation

$$\mathbf{AX} = \mathbf{Y}$$

where **A** is 3 x 3 matrix, **X** and **Y** are 3 x 1 matrices with

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$ **[3 marks]**

(ii) Let $\mathbf{B} = \begin{pmatrix} -4 & 0 & 2 \\ 0 & 6 & -4 \\ 2 & -4 & 2 \end{pmatrix}$.

a) Calculate **AB**. **[3 marks]**

b) Deduce the inverse $\mathbf{A}^{-1}$ of **A**. **[3 marks]**

(iii) Hence, or otherwise, determine the number of cars and buses used in the 34 km tours. **[4 marks]**

Total 25 marks

END OF TEST

TEST CODE **22234032**

MAY/JUNE 2008

FORM TP 2008244

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

27 JUNE 2008 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2008**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) Given that $x = \ln [y + \sqrt{(y^2 - 1)}]$, $y > 1$, express y in terms of x. **[5 marks]**

(b) Use the substitution $u = \sin x$ to find

$$\int \cos^3 x \, dx.$$ **[6 marks]**

(c) Engine oil at temperature **T** °C cools according to the model

$$T = 60\, e^{-kt} + 10$$

where t is the time in minutes from the moment the engine is switched off.

(i) Determine the initial temperature of the oil when the engine is **first** switched off. **[2 marks]**

(ii) If the oil cools to 32°C after three minutes, determine how long it will take for the oil to cool to a temperature of 15°C. **[7 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Write the general term of the series whose first four terms are

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \frac{1}{5 \times 7} + \frac{1}{7 \times 9} + \ldots$$ **[2 marks]**

(ii) Use the method of differences to find the sum of the first n terms. **[5 marks]**

(iii) Show that the series converges and find its sum to infinity. **[3 marks]**

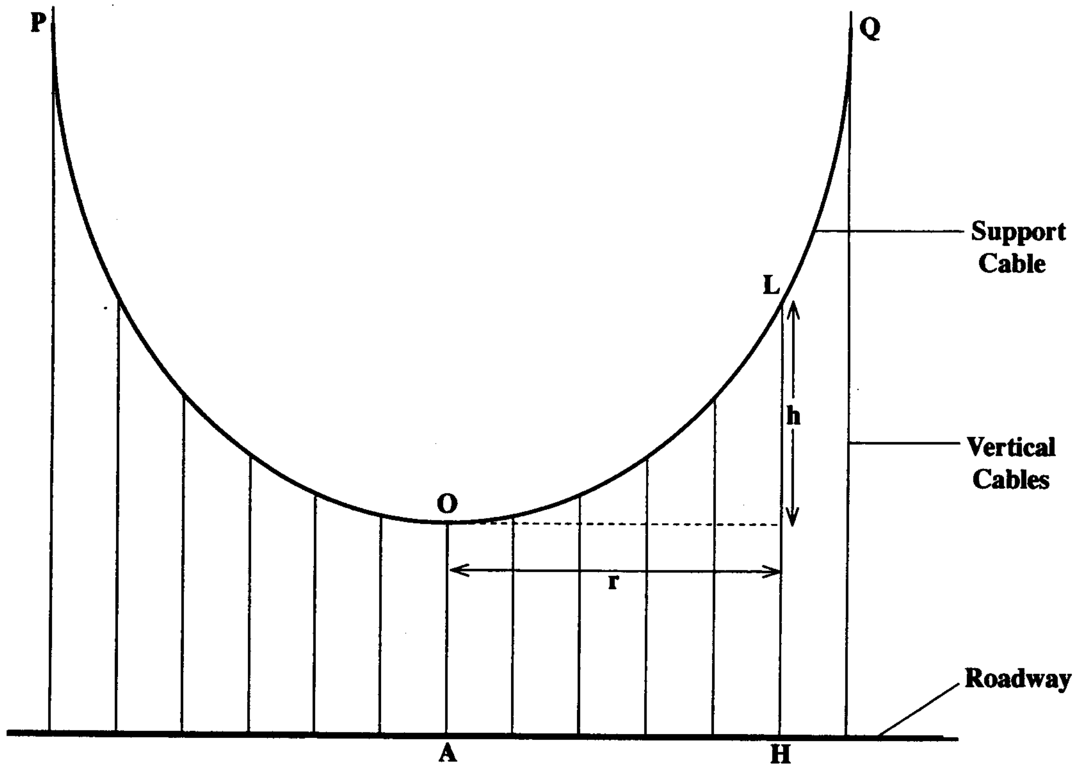
GO ON TO THE NEXT PAGE

(b) The diagram below **(not drawn to scale)** shows part of the suspension of a bridge. A support cable POQ, is in the shape of a curve with equation

$$y = \frac{1}{10}\left| x^{3/2} \right| + c, \text{ where } c \text{ is a constant.}$$

Starting at P, through O and finishing at Q, 51 vertical cables are bolted 1 metre apart to the roadway and to the support cable POQ. The shortest vertical cable OA has a length of 5 metres, where O is the lowest point of the support cable.

The cost, in dollars, of installing the cable LH at a horizontal distance of r metres from OA is \$100 plus \$ $h\sqrt{r}$, where h is the height of the point L above O.



(i) Find, in terms of r, the cost of installing the cable LH. **[4 marks]**

(ii) Hence, obtain the **total** cost of installing the 51 vertical cables. **[6 marks]**

Total 20 marks

GO ON TO THE NEXT PAGE

SECTION C (Module 3)

Answer this question.

3. (a) Let $z = \dfrac{(1-2i)(7+i)}{(1+i)^2}$.

(i) Express z in the form $a + bi$, where $a, b \in \mathbf{R}$. **[5 marks]**

(ii) Calculate the **exact** value of $|z|$. **[3 marks]**

(b) Two 3 x 1 matrices **X** and **Y** satisfy the equation **X** = **AY**, where the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 3 & 2 & 4 \\ 4 & 1 & 6 \end{pmatrix} \text{ is non-singular.}$$

Find

(i) $\mathbf{A}^{-1}$ **[8 marks]**

(ii) **Y**, when $\mathbf{X} = \begin{pmatrix} 6 \\ 4 \\ 11 \end{pmatrix}$. **[4 marks]**

Total 20 marks

END OF TEST

838244

TEST CODE **02234020**

FORM TP 2009237

MAY/JUNE 2009

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

27 MAY 2009 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 5 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Find $\frac{dy}{dx}$ if

(i) $y = \sin^2 5x + \sin^2 3x + \cos^2 3x$ **[3 marks]**

(ii) $y = \sqrt{\cos x^2}$ **[4 marks]**

(iii) $y = x^x$. **[4 marks]**

(b) (i) Given that $y = \cos^{-1} x$, where $0 \leq \cos^{-1} x \leq \pi$, prove that $\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$.

[Note: $\cos^{-1}x \equiv \arccos x$] **[7 marks]**

(ii) The parametric equations of a curve are defined in terms of a parameter t by $y = \sqrt{1-t}$ and $x = \cos^{-1} t$, where $0 \leq t < 1$.

a) Show that $\frac{dy}{dx} = \frac{\sqrt{1+t}}{2}$. **[4 marks]**

b) Hence, find $\frac{d^2y}{dx^2}$ in terms of t, giving your answer in simplified form. **[3 marks]**

Total 25 marks

2. (a) Sketch the region whose area is defined by the integral $\int_0^1 \sqrt{1-x^2}\, dx$. **[3 marks]**

(b) Using FIVE vertical strips, apply the trapezium rule to show that $\int_0^1 \sqrt{1-x^2}\, dx \approx 0.759$. **[6 marks]**

(c) (i) Use integration by parts to show that, if $I = \int \sqrt{1-x^2}\, dx$, then

$$I = x\sqrt{1-x^2} - I + \int \frac{1}{\sqrt{1-x^2}}\, dx.$$ **[9 marks]**

GO ON TO THE NEXT PAGE

(ii) Deduce that $I = \dfrac{x\sqrt{1-x^2} + \sin^{-1} x}{2} + c$, where c is an arbitrary constant of integration.

[Note: $\cos^{-1}x \equiv \arccos x$] **[2 marks]**

(iii) Hence, find $\int_0^1 \sqrt{1-x^2}\ dx$. **[3 marks]**

(iv) Use the results in Parts (b) and (c) (iii) above to find an approximation to π. **[2 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) A sequence $\{t_n\}$ is defined by the recurrence relation

$$t_{n+1} = t_n + 5,\ t_1 = 11 \text{ for all } n \in \mathbf{N}.$$

(i) Determine t_2, t_3 and t_4. **[3 marks]**

(ii) Express t_n in terms of n. **[5 marks]**

(b) Find the range of values of x for which the common ratio r of a convergent geometric series is $\dfrac{2x-3}{x+4}$. **[8 marks]**

(c) Let $f(r) = \dfrac{1}{r+1}$, $r \in \mathbf{N}$.

(i) Express $f(r) - f(r+1)$ in terms of r. **[3 marks]**

(ii) Hence, or otherwise, find

$$S_n = \sum_{r=1}^{n} \frac{4}{(r+1)(r+2)}.$$ **[4 marks]**

(iii) Deduce the sum to infinity of the series in (c) (ii) above. **[2 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

4. (a) (i) Find $n \in \mathrm{N}$ such that $5({}^{n}C_2) = 2({}^{n+2}C_2)$. **[5 marks]**

(ii) The coefficient of x^2 in the expansion of

$$(1 + 2x)^5 \ (1 + px)^4$$

is –26. Find the possible values of the real number p. **[7 marks]**

(b) (i) Write down the first FOUR non-zero terms of the power series expansion of $\ln(1 + 2x)$, stating the range of values of x for which the series is valid. **[2 marks]**

(ii) Use Maclaurin's theorem to obtain the first THREE non-zero terms in the power series expansion in x of $\sin 2x$. **[7 marks]**

(iii) Hence, or otherwise, obtain the first THREE non-zero terms in the power series expansion in x of

$$\ln(1 + \sin 2x).$$ **[4 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) A committee of 4 persons is to be chosen from 8 persons, including Mr Smith and his wife. Mr Smith will not join the committee without his wife, but his wife will join the committee without him.

Calculate the number of ways in which the committee of 4 persons can be formed. **[5 marks]**

(b) Two balls are drawn without replacement from a bag containing 12 balls numbered 1 to 12.

Find the probability that

(i) the numbers on BOTH balls are even **[4 marks]**

(ii) the number on one ball is odd and the number on the other ball is even. **[4 marks]**

(c) (i) Find complex numbers $u = x + iy$ such that x and y are real numbers and

$$u^2 = -15 + 8i.$$ **[7 marks]**

(ii) Hence, or otherwise, solve for z the equation

$$z^2 - (3 + 2i)\,z + (5 + i) = 0.$$ **[5 marks]**

Total 25 marks

6. (a) Solve for x the equation

$$\begin{vmatrix} x-3 & 1 & -1 \\ 1 & x-5 & 1 \\ -1 & 1 & x-3 \end{vmatrix} = 0.$$ **[10 marks]**

(b) (i) Given the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 3 & 9 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 30 & -12 & 2 \\ 5 & -8 & 3 \\ -5 & 4 & 1 \end{pmatrix},$$

a) find **AB** **[4 marks]**

b) hence deduce the inverse $\mathbf{A}^{-1}$ of the matrix **A**. **[3 marks]**

(ii) A system of equations is given by

$$x - y + z = 1$$

$$x - 2y + 4z = 5$$

$$x + 3y + 9z = 25.$$

a) Express the system in the form

$\mathbf{A}x = \mathbf{b}$, where **A** is a matrix and x and **b** are column vectors.

b) Hence, or otherwise, solve the system of equations. **[5 marks]**

Total 25 marks

END OF TEST

TEST CODE **02234032**

FORM TP 2009238

MAY/JUNE 2009

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

03 JUNE 2009 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x(x+1)} = (x+1)e^{-x^2}.$$

[7 marks]

(b) A curve is being cut by an automatic machine. The x and y coordinates of the curve are connected by the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 5\sin x + 3\cos x.$$

Find the equation of the curve, given that the curve passes through the origin and that $y = e^{-\pi} - e^{4\pi}$ when $x = \pi$. **[13 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) Prove by mathematical induction that

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

for all integers $n \geq 1$. **[7 marks]**

(b) An A.P. with ten terms has first term 60 and last term –120. Find the sum of ALL the terms. **[4 marks]**

(c) John's starting annual salary at a company is \$25 000. His contract at the company states that his annual salary in subsequent years will be increased by 2% over the salary of the previous year.

Find, **to the nearest dollar,**

(i) John's salary for the tenth year with the company **[4 marks]**

(ii) the TOTAL amount of money which the company would have paid to John at the end of his first ten years with the company. **[5 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) There are 6 staff members and 7 students on the sports council of a college. A committee of 10 persons is to be selected to organize a tournament. Calculate the number of ways in which the committee can be selected if the number of students must be **greater than or equal to** the number of staff members. **[6 marks]**

(b) A and B are events such that

$$P(A) = 0.6, \quad P(B) = 0.2 \quad \text{and} \quad P(A \cap B) = 0.1.$$

Calculate

(i) $P(A \cup B)$ **[2 marks]**

(ii) $P(A \cap B')$ **[2 marks]**

(iii) the probability that exactly ONE of A and B will occur. **[4 marks]**

(c) (i) Show that the locus of the complex number z such that

$$|z + \mathrm{i} - 1| = 5$$

is a circle. **[4 marks]**

(ii) Find the centre and radius of the circle in (c) (i) above. **[2 marks]**

Total 20 marks

END OF TEST

TEST CODE **02234020**

FORM TP 2010230

MAY/JUNE 2010

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

26 MAY 2010 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

<u>INSTRUCTIONS TO CANDIDATES</u>

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

<u>Examination Materials Permitted</u>

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) The temperature of water, $x°$ C, in an insulated tank at time, t hours, may be modelled by the equation $x = 65 + 8e^{-0.02t}$. Determine the

(i) initial temperature of the water in the tank **[2 marks]**

(ii) temperature at which the water in the tank will eventually stabilize **[2 marks]**

(iii) time when the temperature of the water in the tank is 70° C. **[4 marks]**

(b) (i) Given that $y = e^{\tan^{-1}(2x)}$, where $-\frac{1}{2}\pi < \tan^{-1}(2x) < \frac{1}{2}\pi$, show that

$$(1 + 4x^2)\frac{dy}{dx} = 2y.$$ **[4 marks]**

(ii) Hence, show that $(1 + 4x^2)^2 \frac{d^2y}{dx^2} = 4y(1 - 4x)$. **[4 marks]**

(c) Determine $\int \frac{4}{e^x + 1}\,dx$

(i) by using the substitution $u = e^x$ **[6 marks]**

(ii) by first multiplying both the numerator and denominator of the integrand $\frac{4}{e^x + 1}$ by e^{-x} before integrating. **[3 marks]**

Total 25 marks

2. (a) (i) Given that n is a positive integer, find $\frac{d}{dx}[x(\ln x)^n]$. **[4 marks]**

(ii) Hence, or otherwise, derive the reduction formula $I_n = x(\ln x)^n - nI_{n-1}$, where $I_n = \int (\ln x)^n\,dx$. **[4 marks]**

(iii) Use the reduction formula in (a) (ii) above to determine $\int (\ln x)^3\,dx$. **[6 marks]**

(b) The amount of salt, y kg, that dissolves in a tank of water at time t minutes satisfies the differential equation $\frac{dy}{dt} + \frac{2y}{t+10} = 3$.

(i) Using a suitable integrating factor, show that the general solution of this differential equation is $y = t + 10 + \frac{c}{(t+10)^2}$, where c is an arbitrary constant. **[7 marks]**

(ii) Given that the tank initially contains 5 kg of salt in the liquid, calculate the amount of salt that dissolves in the tank of water at $t = 15$. **[4 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The first four terms of a sequence are

$2 \times 3, \quad 5 \times 5, \quad 8 \times 7, \quad 11 \times 9.$

(i) Express, in terms of r, the r th term of the sequence. **[2 marks]**

(ii) If S_n denotes the series formed by summing the first n terms of the sequence, find S_n in terms of n. **[5 marks]**

(b) The 9th term of an A.P. is three times the 3rd term and the sum of the first 10 terms is 110. Find the first term a **and** the common difference d. **[6 marks]**

(c) (i) Use the binominal theorem to expand $(1 + 2x)^{\frac{1}{2}}$ as far as the term in x^3, stating the values of x for which the expansion is valid. **[5 marks]**

(ii) Prove that $\frac{x}{1 + x + \sqrt{1+2x}} = \frac{1}{x}(1 + x - \sqrt{1+2x})$ for $x > 0$. **[4 marks]**

(iii) Hence, or otherwise, show that, if x is small so that the term in x^3 and higher powers of x can be neglected, the expansion in (c) (ii) above is approximately equal to

$\frac{1}{2}x(1 - x)$. **[3 marks]**

Total 25 marks

4. (a) (i) By expressing ${}^nC_r$ and ${}^nC_{r-1}$ in terms of factorials, prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

[6 marks]

(ii) a) Given that r is a positive integer and $f(r) = \frac{1}{r!}$, show that

$$f(r) - f(r+1) = \frac{r}{(r+1)!}$$ **[3 marks]**

b) Hence, or otherwise, find the sum

$$S_n = \sum_{r=1}^{n} \frac{r}{(r+1)!}.$$ **[5 marks]**

c) Deduce the sum to infinity of S_n in (ii) b) above. **[2 marks]**

(b) (i) Show that the function $f(x) = x^3 - 6x + 4$ has a root x in the closed interval $[0, 1]$.

[5 marks]

(ii) By taking 0.6 as a first approximation of x_1 in the interval $[0, 1]$, use the Newton-Raphson method to obtain a second approximation x_2 in the interval $[0, 1]$.

[4 marks]

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Calculate

(i) the number of different permutations of the 8 letters of the word **SYLLABUS** **[3 marks]**

(ii) the number of different selections of 5 letters which can be made from the letters of the word **SYLLABUS**. **[5 marks]**

(b) The events A and B are such that $P(A) = 0.4$, $P(B) = 0.45$ and $P(A \cup B) = 0.68$.

(i) Find $P(A \cap B)$. **[3 marks]**

(ii) Stating a reason in each case, determine whether or not the events A and B are

a) mutually exclusive **[3 marks]**

b) independent. **[3 marks]**

(c) (i) Express the complex number $(2 + 3\text{i}) + \frac{\text{i} - 1}{\text{i} + 1}$ in the form $a + \text{i}b$, where a and b are both real numbers. **[4 marks]**

(ii) Given that $1 - \text{i}$ is the root of the equation $z^3 + z^2 - 4z + 6 = 0$, find the remaining roots. **[4 marks]**

Total 25 marks

6. (a) A system of equations is given by

$$\begin{aligned} x + y + z &= 0 \\ 2x + y - z &= -1 \\ x + 2y + 4z &= k \end{aligned}$$

where k is a real number.

(i) Write the augmented matrix of the system. **[2 marks]**

(ii) Reduce the augmented matrix to echelon form. **[3 marks]**

(iii) Deduce the value of k for which the system is consistent. **[2 marks]**

(iv) Find ALL solutions corresponding to the value of k obtained in (iii) above. **[4 marks]**

(b) Given $A = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

(i) Find

a) A^2 **[4 marks]**

b) $B = 3I + A - A^2$ **[4 marks]**

(ii) Calculate AB. **[4 marks]**

(iii) Deduce the inverse, A^{-1}, of the matrix A. **[2 marks]**

Total 25 marks

END OF TEST

TEST CODE **02234032**

FORM TP 2010231

MAY/JUNE 2010

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

02 JUNE 2010 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2009**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this questions.

1. (a) Express in partial fractions

$$\frac{1 - x^2}{x\,(x^2 + 1)}.$$ **[7 marks]**

(b) The rate of change of a population of bugs is modelled by the differential equation $\frac{dy}{dt} - ky = 0$, where y is the size of the population at time, t, given in days, and k is the constant. Initially, the population is y_0 and it doubles in size in 3 days.

(i) Show that

a) $y = y_0\, e^{kt}$ **[7 marks]**

b) $k = \frac{1}{3} \ln 2.$ **[3 marks]**

(ii) Find the proportional increase in population at the end of the second day. **[3 marks]**

Total 20 marks

GO ON TO THE NEXT PAGE

SECTION B (Module 2)

Answer this questions.

2. (a) The sum to infinity of a convergent geometric series is equal to six times the first term. Find the common ratio of the series. **[5 marks]**

(b) Find the sum to infinity of the series $\sum_{r=1}^{\infty} a_r$ whose r th term a_r is

$$\frac{2r+1}{r!}.$$ **[8 marks]**

(c) A truck bought for \$15 000 depreciates at the rate of $12\frac{1}{2}$% each year. Calculate the value of the truck

(i) after 1 year **[2 marks]**

(ii) after t years **[2 marks]**

(iii) when its value FIRST falls below \$5 000. **[3 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this questions.

3. (a) Find the number of integers between 300 and 1 000 which can be formed by using the digits 1, 3, 5, 7 and 9

(i) if NO digit can be repeated **[3 marks]**

(ii) if ANY digit can be repeated. **[2 marks]**

(b) Find the probability that a number in (a) (ii) above ends with the digit 9. **[3 marks]**

(c) A farmer made three separate visits to the chicken farm to purchase chickens. On each visit he paid \$ x for each grade A chicken, \$ y for each grade B chicken and \$ z for each grade C. His calculations are summarised in the table below.

Number of Visits	Number of Chickens Bought			Total Spent \$
	Grade A	Grade B	Grade C	
1	20	40	60	1 120
2	40	60	80	1 720
3	60	80	120	2 480

(i) Use the information above to form a system of linear equations in x, y and z. **[3 marks]**

(ii) Express the system of equations in the form $\mathbf{A}\mathbf{x} = \mathbf{b}$. **[2 marks]**

(iii) Solve the equations to find x, y and z. **[7 marks]**

Total 20 marks

END OF TEST

FORM TP 2011234

TEST CODE **02234020**

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 ½ hours

25 MAY 2011 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Find $\frac{dy}{dx}$ if

(i) $x^2 + y^2 - 2x + 2y - 14 = 0$ **[3 marks]**

(ii) $y = e^{\cos x}$ **[3 marks]**

(iii) $y = \cos^2 6x + \sin^2 8x.$ **[3 marks]**

(b) Let $y = x \sin \frac{1}{x}$, $x \neq 0.$

Show that

(i) $x \frac{dy}{dx} = y - \cos\left(\frac{1}{x}\right)$ **[3 marks]**

(ii) $x^4 \frac{d^2y}{dx^2} + y = 0.$ **[3 marks]**

(c) A curve is given by the parametric equations $x = \sqrt{t}$, $y - t = \frac{1}{\sqrt{t}}$.

(i) Find the gradient of the tangent to the curve at the point where $t = 4$. **[7 marks]**

(ii) Find the equation of the tangent to the curve at the point where $t = 4$. **[3 marks]**

Total 25 marks

2. (a) Let $F_n(x) = \frac{1}{n!}\int_0^x t^n e^{-t}\, dt$.

(i) Find $F_0(x)$ and $F_n(0)$, given that $0! = 1$. **[3 marks]**

(ii) Show that $F_n(x) = F_{n-1}(x) - \frac{1}{n!} x^n e^{-x}$. **[7 marks]**

(iii) Hence, show that if M is an integer greater than 1, then

$$e^x F_M(x) = -\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^M}{M!}\right) + (e^x - 1).$$ **[4 marks]**

(b) (i) Express $\frac{2x^2 + 3}{(x^2 + 1)^2}$ in partial fractions. **[5 marks]**

(ii) Hence, find $\int \frac{2x^2 + 3}{(x^2 + 1)^2}\, dx$. **[6 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The sequence of positive terms, $\{x_n\}$, is defined by $x_{n+1} = x_n^2 + \frac{1}{4}$, $x_1 < \frac{1}{2}$, $n \geq 1$.

(i) Show, by mathematical induction, that $x_n < \frac{1}{2}$ for all positive integers n. **[5 marks]**

(ii) By considering $x_{n+1} - x_n$, show that $x_n < x_{n+1}$. **[3 marks]**

(b) (i) Find the constants A and B such that

$$\frac{2-3x}{(1-x)(1-2x)} \equiv \frac{A}{1-x} + \frac{B}{1-2x}.$$ **[3 marks]**

(ii) Obtain the **first** FOUR non-zero terms of the expansion of each of $(1-x)^{-1}$ and $(1-2x)^{-1}$ as power series of x in ascending order. **[4 marks]**

(iii) Find

a) the range of values of x for which the series expansion of

$$\frac{2-3x}{(1-x)(1-2x)}$$

is valid **[2 marks]**

b) the coefficient of x^n in (iii) a) above. **[2 marks]**

(iv) The sum, S_n, of the first n terms of a series is given by

$$S_n = n(3n-4).$$

Show that the series is an Arithmetic Progression (A.P.) with common difference 6. **[6 marks]**

Total 25 marks

4. (a) A Geometric Progression (G.P.) with first term a and common ratio r, $0 < r < 1$, is such that the sum of the first three terms is $\frac{26}{3}$ and their product is 8.

(i) Show that $r + 1 + \frac{1}{r} = \frac{13}{3}$. **[4 marks]**

(ii) Hence, find

a) the value of r **[4 marks]**

b) the value of a **[1 mark]**

c) the sum to infinity of the G.P. **[2 marks]**

(b) Expand

$$\frac{2}{e^x + e^{-x}}, \ |x| < 1$$

in ascending powers of x as far as the term in x^4. **[5 marks]**

(c) Let $f(r) = \frac{1}{r(r+1)}, r \in \mathbf{N}$.

(i) Express $f(r) - f(r+1)$ in terms of r. **[3 marks]**

(ii) Hence, or otherwise, find

$$S_n = \sum_{r=1}^{n} \frac{3}{r(r+1)(r+2)}.$$ **[4 marks]**

(iii) Deduce the sum to infinity of the series in (c) (ii) above. **[2 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) $\begin{pmatrix} n \\ r \end{pmatrix}$ is defined as the number of ways of selecting r distinct objects from a given set of n distinct objects. From the definition, show that

(i) $\begin{pmatrix} n \\ r \end{pmatrix} = \begin{pmatrix} n \\ n-r \end{pmatrix}$ **[2 marks]**

(ii) $\begin{pmatrix} n+1 \\ r \end{pmatrix} = \begin{pmatrix} n \\ r \end{pmatrix} + \begin{pmatrix} n \\ r-1 \end{pmatrix}$ **[4 marks]**

(iii) Hence, prove that

$$\left(\begin{pmatrix} 8 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ 5 \end{pmatrix}\right) \times \left(\begin{pmatrix} 8 \\ 3 \end{pmatrix} + \begin{pmatrix} 8 \\ 2 \end{pmatrix}\right)$$

is a perfect square. **[3 marks]**

(b) (i) Find the number of 5-digit numbers greater than 30 000 which can be formed with the digits, 1, 3, 5, 6 and 8, if no digit is repeated. **[3 marks]**

(ii) What is the probability of one of the numbers chosen in (b) (i) being even? **[5 marks]**

(c) (i) a) Show that $(1 - i)$ is one of the square roots of $-2i$. **[2 marks]**

b) Find the other square root. **[1 mark]**

(ii) Hence, find the roots of the quadratic equation

$$z^2 - (3 + 5i)\, z + (8i - 4) = 0.$$ **[5 marks]**

Total 25 marks

6. (a) The matrix $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -3 & 2 \\ -1 & -3 & -2 \end{pmatrix}$.

(i) Show that $|\mathbf{A}| = 5$. **[3 marks]**

(ii) Matrix **A** is changed to form new matrices **B**, **C** and **D**. Write down the determinant of EACH of the new matrices, **giving a reason for your answer in EACH case**.

a) Matrix **B** is formed by interchanging row 1 and row 2 of matrix **A** and then interchanging column 1 and column 2 of the resulting matrix. **[2 marks]**

b) Row 1 of matrix **C** is formed by adding row 2 to row 1 of matrix **A**. The other rows remain unchanged. **[2 marks]**

c) Matrix **D** is formed by multiplying each element of matrix **A** by 5. **[2 marks]**

(b) Given the matrix $\mathbf{M} = \begin{pmatrix} 12 & -1 & 5 \\ 2 & -1 & 0 \\ -9 & 2 & -5 \end{pmatrix}$,

Find

(i) **AM** **[3 marks]**

(ii) the inverse, $\mathbf{A}^{-1}$, of **A**. **[2 marks]**

(c) (i) Write the system of equations

$$\begin{aligned} x + y + z &= 5 \\ 2x - 3y + 2z &= -10 \\ -x - 3y - 2z &= -11 \end{aligned}$$

in the form $\mathbf{A}x = \mathbf{b}$. **[1 mark]**

(ii) Show that $x = \mathbf{A}^{-1}\mathbf{b}$. **[2 marks]**

(iii) Hence, solve the system of equations. **[2 marks]**

(iv) a) Show that $(x, y, z) = (1, 1, 1)$ is a solution of the following system of equations:

$$\begin{aligned} x + y + z &= 3 \\ 2x + 2y + 2z &= 6 \\ 3x + 3y + 3z &= 9 \end{aligned}$$

[1 mark]

b) Hence, find the general solution of the system. **[5 marks]**

Total 25 marks

END OF TEST

TEST CODE **02234032**

FORM TP 2011235

MAY/JUNE 2011

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – PAPER 03/B

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 ½ hours

01 JUNE 2011 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) A target is moving along a curve whose parametric equations are

$$x = 4 - 3\cos t,\ \ y = 5 + 2\sin t,$$

where t is the time. The distances are measured in metres.

Let θ be the angle which the tangent to the curve makes with the positive x-axis.

(i) Find the rate at which θ is increasing or decreasing when $t = \frac{2\pi}{3}$ seconds. **[7 marks]**

(ii) What are the units of the rate of increase? **[1 mark]**

(iii) Find the Cartesian equation of the curve. **[2 marks]**

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 8x^2.$$ **[10 marks]**

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Show that the equation $x^2 + 8x - 8 = 0$ has a root, α, in the interval $[0, 1]$. **[3 marks]**

(ii) By taking $x_0 = 0$ as the first approximation for α and using the formula $x_{n+1} = \frac{8 - x_n^{\ 2}}{8}$ **three** times, find a better approximation for α. **[3 marks]**

(b) (i) Write down the **first** FOUR non-zero terms of the expansions of $\ln(1 - x)$ and e^{-x} in ascending powers of x, stating for EACH expansion the range of values of x for which it is valid. **[3 marks]**

(ii) If $-1 \leq x < 1$ and $y = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$, prove that

$$x = 1 - e^{-y}.$$ **[2 marks]**

(c) In a model for the growth of a population, p_n is the number of individuals in the population **at the end** of n years. Initially, the population consists of 1000 individuals. In EACH calendar year (January to December), the population increases by 20% and on 31 December, 100 individuals leave the population.

(i) Calculate the values of p_1 and p_2. **[2 marks]**

(ii) Obtain an equation connecting p_{n+1} and p_n. **[1 mark]**

(iii) Show that $p_n = 500(1.2)^n + 500$. **[6 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) Let $\mathbf{A} = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$

(i) Show that $\mathbf{A}^2 - 3\mathbf{A} + 2\mathbf{I} = 0$. **[6 marks]**

(ii) Deduce that $\mathbf{A}^{-1} = \frac{1}{2}(3\mathbf{I} - \mathbf{A})$. **[4 marks]**

(iii) Hence, find the solution of the system of equations

$$\begin{aligned} 5x - 6y - 6z &= 10 \\ -x + 4y + 2z &= -4 \\ 3x - 6y - 4z &= 8. \end{aligned}$$

[3 marks]

(b) If $z = \frac{2+i}{1-i}$, find the real and imaginary parts of $z + \frac{1}{z}$. **[4 marks]**

(c) If $z + \frac{1}{z}$ is written in the form $r(\cos\theta + i\sin\theta)$ where r is the real and positive, find r and $\tan\theta$. **[3 marks]**

Total 20 marks

END OF TEST

TEST CODE **02234020**

FORM TP 2012234

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

25 MAY 2012 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 7 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer BOTH questions.

1. (a) (i) Given the curve $y = x^2 e^x$,

a) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **[5 marks]**

b) find the ***x*-coordinates** of the points at which $\frac{dy}{dx} = 0$ **[2 marks]**

c) find the ***x*-coordinates** of the points at which $\frac{d^2y}{dx^2} = 0$ **[2 marks]**

(ii) Hence , determine if the coordinates identified in (i) b) and c) above are at the maxima, minima or points of inflection of $y = x^2 e^x$. **[7 marks]**

(b) A curve is defined by the parametric equations $x = \sin^{-1} \sqrt{t}$, $y = t^2 - 2t$.

Find

(i) the gradient of a tangent to the curve at the point with parameter t **[6 marks]**

(ii) the equation of the tangent at the point where $t = \frac{1}{2}$. **[3 marks]**

Total 25 marks

2. (a) (i) Express

$$\frac{x^2 - 3x}{(x-1)(x^2+1)} \text{ in partial fractions.}$$ **[7 marks]**

(ii) Hence, find

$$\int \frac{x^2 - 3x}{x^3 - x^2 + x - 1}\, \mathrm{d}x.$$ **[5 marks]**

(b) (i) Given that $\sin A \cos B - \cos A \sin B = \sin(A - B)$ show that

$$\cos 3x \sin x = \sin 3x \cos x - \sin 2x.$$ **[2 marks]**

(ii) If $I_m = \int \cos^m x \sin 3x \, \mathrm{d}x$ and

$$J_m = \int \cos^m x \sin 2x \, \mathrm{d}x,$$

prove that $(m + 3)\, I_m = mJ_{m-1} - \cos^m x \cos 3x$. **[7 marks]**

(iii) Hence, by putting $m = 1$, prove that

$$4\int_0^{\frac{\pi}{4}} \cos x \sin 3x \, \mathrm{d}x = \int_0^{\frac{\pi}{4}} \sin 2x \, \mathrm{d}x + \frac{3}{2}.$$ **[2 marks]**

(iv) Evaluate $\int_0^{\frac{\pi}{4}} \sin 2x \, \mathrm{d}x$. **[2 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) For a particular G.P., $u_6 = 486$ and $u_{11} = 118\ 098$, where u_n is the n^{th} term.

(i) Calculate the first term, a, and the common ratio, r. **[5 marks]**

(ii) Hence, calculate n if $S_n = 177\ 146$. **[4 marks]**

(b) The first four terms of a sequence are $1 \times 3, 2 \times 4, 3 \times 5, 4 \times 6$.

(i) Express, in terms of r, the r^{th} term, u_r, of the sequence. **[2 marks]**

(ii) Prove, by mathematical induction, that

$$\sum_{r=1}^{n} u_r = \frac{1}{6} n\ (n+1)\ (2n+7),\ \forall n \in \mathbf{N}.$$ **[7 marks]**

(c) (i) Use Maclaurin's Theorem to find the first three non-zero terms in the power series expansion of $\cos 2x$. **[5 marks]**

(ii) Hence, or otherwise, obtain the first two non-zero terms in the power series expansion of $\sin^2 x$. **[2 marks]**

Total 25 marks

4. (a) (i) Express $\binom{n}{r}$ in terms of factorials. **[1 mark]**

(ii) Hence, show that $\binom{n}{r} = \binom{n}{n-r}$. **[3 marks]**

(iii) Find the coefficient of x^4 in $\left(x^2 - \frac{3}{x}\right)^8$. **[5 marks]**

(iv) Using the identity $(1 + x)^{2n} = (1 + x)^n (1 + x)^n$, show that

$$\binom{2n}{n} = c_0^2 + c_1^2 + c_3^2 + \ldots + c_{n-1}^2 + c_n^2, \text{ where } c_r = \binom{n}{r}.$$

[8 marks]

(b) Let $f(x) = 2x^3 + 3x^2 - 4x - 1 = 0$.

(i) Use the intermediate value theorem to determine whether the equation $f(x)$ has any roots in the interval [0.2, 2]. **[2 marks]**

(ii) Using $x_1 = 0.6$ as a first approximation of a root **T** of $f(x)$, execute FOUR iterations of the Newton–Raphson method to obtain a second approximation, x_2, of **T**. **[6 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C (Module 3)

Answer BOTH questions.

5. (a) How many 4-digit even numbers can be formed from the digits 1, 2, 3, 4, 6, 7, 8

(i) if each digit appears at most once? **[4 marks]**

(ii) if there is no restriction on the number of times a digit may appear? **[3 marks]**

(b) A committee of five is to be formed from among six Jamaicans, two Tobagonians and three Guyanese.

(i) Find the probability that the committee consists entirely of Jamaicans. **[3 marks]**

(ii) Find the number of ways in which the committee can be formed, given the following restriction: *There are as many Tobagonians on the committee as there are Guyanese.* **[6 marks]**

(c) Let **A** be the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.

(i) Find the matrix **B**, where $\mathbf{B} = \mathbf{A}^2 - 3\mathbf{A} - \mathbf{I}$. **[3 marks]**

(ii) Show that $\mathbf{AB} = -9\mathbf{I}$. **[1 mark]**

(iii) Hence, find the inverse, $\mathbf{A}^{-1}$, of **A**. **[2 marks]**

(iv) Solve the system of linear equations

$$\mathbf{B}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}.$$

[3 marks]

Total 25 marks

6. (a) (i) Draw the points A and B on an Argand diagram,

$$\text{where } A = \frac{1+i}{1-i} \text{ and } B = \frac{\sqrt{2}}{1-i}.$$ **[6 marks]**

(ii) Hence, or otherwise, show that the argument of $\dfrac{(1+\sqrt{2}+i)}{1-i}$ is EXACTLY $\dfrac{3\pi}{8}$.

[5 marks]

(b) (i) Find ALL complex numbers, z, such that $z^2 = i$. **[3 marks]**

(ii) Hence, find ALL complex roots of the equation

$$z^2 - (3+5i)z - (4-7i) = 0.$$ **[5 marks]**

(c) Use de Moivre's theorem to show that

$$\cos 6\theta = \cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta.$$ **[6 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

TEST CODE **02234032**

FORM TP 2012235

MAY/JUNE 2012

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2 – Paper 032

ANALYSIS, MATRICES AND COMPLEX NUMBERS

1 hour 30 minutes

01 JUNE 2012 (a.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

<u>Examination Materials Permitted</u>

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A (Module 1)

Answer this question.

1. (a) Given that $y = \dfrac{x\,(x-1)^{\frac{1}{3}}}{1+\sin^3 x}$,

by taking logarithms of both sides, or otherwise, find $\dfrac{dy}{dx}$ in terms of x. **[4 marks]**

(b) (i) Sketch the curve $y = \sqrt{1+x^3}$, for values of x between $-\dfrac{1}{2}$ and 1. **[3 marks]**

(ii) Using the trapezium rule, with 5 intervals, find an approximation to

$$\int_0^1 \sqrt{1+x^3}\,dx.$$ **[5 marks]**

(c) (i) Use integration by parts to find

$$\int x^2 \cos x\,dx.$$ **[6 marks]**

(ii) Hence, find the area under the curve $y = x^2 \cos x$, between $x = 0$ and $x = \dfrac{\pi}{2}$.

[2 marks]

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) (i) Write down the binomial expansion of $\left(1+\frac{1}{4}x\right)^5$. **[4 marks]**

(ii) Hence, calculate $(1.025)^5$ correct to three decimal places. **[4 marks]**

(b) Let $f(x) = x^2 - 5x + 3$ and $g(x) = e^x$ be two functions.

(i) Sketch the graphs for $f(x)$ and $g(x)$ on the **same** coordinate axes for the domain $-1 \leq x \leq 2$. **[4 marks]**

(ii) Using $x_1 = 0.3$ as an initial approximation to the root x of $f(x) - g(x) = 0$, execute TWO iterations of the Newton-Raphson method to obtain a better approximation, x_3, of x **correct to four decimal places**. **[6 marks]**

(iii) Assuming that x_3 is the true root of $f(x) - g(x) = 0$, calculate the relative error of x_1. **[2 marks]**

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) A computer programmer is trying to break into a company's code. His program generates a list of all permutations of any set of letters that it is given, without regard for duplicates. For example, given the letters **TTA**, it will generate a list of six 3-letter permutations (words).

If the program generates a list of all 8-letter permutations of **TELESTEL**, without regard for duplicates,

(i) how many times will any given word be repeated in the list? **[5 marks]**

(ii) in how many words will the first four letters be all different? **[5 marks]**

(b) (i) Find the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{pmatrix}.$$

[5 marks]

(ii) Find a 3×1 matrix, $\mathbf{Y}$, such that

$$\mathbf{A} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \mathbf{Y}.$$

[2 marks]

(iii) Hence, find a 3×3 matrix $\mathbf{B}$ such that

$$\mathbf{BY} = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}.$$

[3 marks]

Total 20 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

2013

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Calculate the gradient of the curve $\ln(x^2y) - \sin y = 3x - 2y$ at the point $(1, 0)$. **[5 marks]**

(b) Let $f(x, y, z) = 3yz^2 - e^{4x}\cos 4z - 3y^2 - 4 = 0$.

Given that $\dfrac{\partial z}{\partial y} = -\dfrac{\partial f/\partial y}{\partial f/\partial z}$, determine $\dfrac{\partial z}{\partial y}$ in terms of x, y and z. **[5 marks]**

(c) Use de Moivre's theorem to prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$ **[6 marks]**

(d) (i) Write the complex number $z = (-1 + i)^7$ in the form $re^{i\theta}$, where $r = |z|$ and $\theta = \arg z$. **[3 marks]**

(ii) Hence, prove that $(-1 + i)^7 = -8(1 + i)$. **[6 marks]**

Total 25 marks

2. (a) (i) Determine $\int \sin x \cos 2x \, dx$. **[5 marks]**

(ii) Hence, calculate $\int_0^{\frac{\pi}{2}} \sin x \cos 2x \, dx$. **[2 marks]**

(b) Let $f(x) = x\,|x| = \begin{cases} x^2 & ; x \geq 0 \\ -x^2 & ; x < 0 \end{cases}$.

Use the trapezium rule with four intervals to calculate the area between $f(x)$ and the x-axis for the domain $-0.75 \leq x \leq 2.25$. **[5 marks]**

(c) (i) Show that $\dfrac{2x^2 + 4}{(x^2 + 4)^2} = \dfrac{2}{x^2 + 4} - \dfrac{4}{(x^2 + 4)^2}$. **[6 marks]**

(ii) Hence, find $\int \dfrac{2x^2 + 4}{(x^2 + 4)^2} dx$. Use the substitution $x = 2\tan\theta$. **[7 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The sequence $\{a_n\}$ is defined by $a_1 = 1, \ a_{n+1} = 4 + 2\sqrt[3]{a_n}$.

Use mathematical induction to prove that $1 \leq a_n \leq 8$ for all n in the set of positive integers. **[6 marks]**

(b) Let $k > 0$ and let $f(k) = \frac{1}{k^2}$.

(i) Show that

a) $f(k) - f(k+1) = \dfrac{2k+1}{k^2\,(k+1)^2}$. **[3 marks]**

b) $\displaystyle\sum_{k=1}^{n} \left(\frac{1}{k^2} - \frac{1}{(k+1)^2} \right) = 1 - \frac{1}{(n+1)^2}$. **[5 marks]**

(iii) Hence, or otherwise, prove that

$$\sum_{k=1}^{\infty} \frac{2k+1}{k^2\,(k+1)^2} = 1.$$ **[3 marks]**

(c) (i) Obtain the first four non-zero terms of the Taylor Series expansion of $\cos x$ in ascending powers of $(x - \frac{\pi}{4})$. **[5 marks]**

(ii) Hence, calculate an approximation to $\cos(\frac{\pi}{16})$. **[3 marks]**

Total 25 marks

4. (a) (i) Obtain the binomial expansion of

$$\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$$

up to the term containing x^2. **[4 marks]**

(ii) Hence, by letting $x = \frac{1}{16}$, compute an approximation of $\sqrt[4]{17} + \sqrt[4]{15}$ to four decimal places. **[4 marks]**

(b) Show that the coefficient of the x^5 term of the product $(x+2)^5(x-2)^4$ is 96. **[7 marks]**

(c) (i) Use the Intermediate Value Theorem to prove that $x^3 = 25$ has at least one root in the interval [2, 3]. **[3 marks]**

(ii) The table below shows the results of the first four iterations in the estimation of the root of $f(x) = x^3 - 25 = 0$ using interval bisection.

The procedure used $a = 2$ and $b = 3$ as the starting points and p_n is the estimate of the root for the n^{th} iteration.

n	a_n	b_n	p_n	$f(p_n)$
1	2	3	2.5	–9.375
2	2.5	3	2.75	–4.2031
3	2.75	3	2.875	–1.2363
4	2.875	3	2.9375	0.3474
5				
6				
......				
......				

Complete the table to obtain an approximation of the root of the equation $x^3 = 25$ correct to 2 decimal places. **[7 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Three letters from the word BRIDGE are selected one after the other without replacement. When a letter is selected, it is classified as either a vowel (V) or a consonant (C).

Use a tree diagram to show the possible outcomes (vowel or consonant) of the THREE selections. Show all probabilities on the diagram. **[7 marks]**

(b) (i) The augmented matrix for a system of three linear equations with variables x, y and z respectively is

$$A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ -5 & 1 & 1 & 2 \\ 1 & -5 & 3 & 3 \end{array}\right)$$

By reducing the augmented matrix to echelon form, determine whether or not the system of linear equations is consistent. **[5 marks]**

(ii) The augmented matrix for another system is formed by replacing the THIRD row of A in (i) above with $(1\ -5\ \ 5 \mid 3)$.

Determine whether the solution of the new system is unique. **Give a reason for your answer.** **[5 marks]**

(c) A country, X, has three airports (A, B, C). The percentage of travellers that use each of the airports is 45%, 30% and 25% respectively. Given that a traveller has a weapon in his/her possession, the probability of being caught is, 0.7, 0.9 and 0.85 for airports A, B, and C respectively.

Let the event that:

- the traveller is caught be denoted by D, and
- the event that airport A, B, or C is used be denoted by A, B, and C respectively.

(i) What is the probability that a traveller using an airport in Country X is caught with a weapon? **[5 marks]**

(ii) On a particular day, a traveller was caught carrying a weapon at an airport in Country X. What is the probability that the traveller used airport C? **[3 marks]**

Total 25 marks

6. (a) (i) Obtain the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x.$$

[7 marks]

(ii) Hence, given that $y = \frac{15\sqrt{2}\pi^2}{32}$, when $x = \frac{\pi}{4}$, determine the constant of the integration. **[5 marks]**

(b) The general solution of the differential equation

$$y'' + 2y' + 5y = 4 \sin 2t$$

is $y = CF + PI$, where CF is the complementary function and PI is a particular integral.

(i) a) Calculate the roots of

$$\lambda^2 + 2\lambda + 5 = 0, \text{ the auxiliary equation.}$$

[2 marks]

b) Hence, obtain the complementary function (CF), the general solution of

$$y'' + 2y' + 5y = 0.$$

[3 marks]

(ii) Given that the form of the particular integral (PI) is

$$u_p(t) = A \cos 2t + B \sin 2t,$$

Show that $A = -\frac{16}{17}$ and $B = \frac{4}{17}$. **[3 marks]**

(iii) Given that $y(0) = 0.04$ and $y'(0) = 0$, obtain the general solution of the differential equation. **[5 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

2013

SECTION A (Module 1)

Answer BOTH questions.

1. (a) Calculate the gradient of the curve $\ln(x^2y) - \sin y = 3x - 2y$ at the point (1, 0). **[5 marks]**

(b) Let $f(x, y, z) = 3yz^2 - e^{4x}\cos 4z - 3y^2 - 4 = 0$.

Given that $\dfrac{\partial z}{\partial y} = -\dfrac{\partial f/\partial y}{\partial f/\partial z}$, determine $\dfrac{\partial z}{\partial y}$ in terms of x, y and z. **[5 marks]**

(c) Use de Moivre's theorem to prove that

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$ **[6 marks]**

(d) (i) Write the complex number $z = (-1 + i)^7$ in the form $re^{i\theta}$, where $r = |z|$ and $\theta = \arg z$. **[3 marks]**

(ii) Hence, prove that $(-1 + i)^7 = -8(1 + i)$. **[6 marks]**

Total 25 marks

2. (a) (i) Determine $\int \sin x \cos 2x \, dx$. **[5 marks]**

(ii) Hence, calculate $\int_0^{\frac{\pi}{2}} \sin x \cos 2x \, dx$. **[2 marks]**

(b) Let $f(x) = x\,|x| = \begin{cases} x^2 & ; x \geq 0 \\ -x^2 & ; x < 0 \end{cases}$.

Use the trapezium rule with four intervals to calculate the area between $f(x)$ and the x-axis for the domain $-0.75 \leq x \leq 2.25$. **[5 marks]**

(c) (i) Show that $\dfrac{2x^2 + 4}{(x^2 + 4)^2} = \dfrac{2}{x^2 + 4} - \dfrac{4}{(x^2 + 4)^2}$. **[6 marks]**

(ii) Hence, find $\int \dfrac{2x^2 + 4}{(x^2 + 4)^2} \, dx$. Use the substitution $x = 2\tan\theta$. **[7 marks]**

Total 25 marks

SECTION B (Module 2)

Answer BOTH questions.

3. (a) The sequence $\{a_n\}$ is defined by $a_1 = 1,\ \ a_{n+1} = 4 + 2\sqrt[3]{a_n}$.

Use mathematical induction to prove that $1 \leq a_n \leq 8$ for all n in the set of positive integers. **[6 marks]**

(b) Let $k > 0$ and let $f(k) = \dfrac{1}{k^2}$.

(i) Show that

a) $f(k) - f(k+1) = \dfrac{2k+1}{k^2\,(k+1)^2}$. **[3 marks]**

b) $\sum\limits_{k=1}^{n} \left(\dfrac{1}{k^2} - \dfrac{1}{(k+1)^2} \right) = 1 - \dfrac{1}{(n+1)^2}$. **[5 marks]**

(iii) Hence, or otherwise, prove that

$$\sum_{k=1}^{\infty} \frac{2k+1}{k^2\,(k+1)^2} = 1.$$ **[3 marks]**

(c) (i) Obtain the first four non-zero terms of the Taylor Series expansion of $\cos x$ in ascending powers of $(x - \frac{\pi}{4})$. **[5 marks]**

(ii) Hence, calculate an approximation to $\cos\left(\frac{\pi}{16}\right)$. **[3 marks]**

Total 25 marks

4. (a) (i) Obtain the binomial expansion of

$$\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$$

up to the term containing x^2. **[4 marks]**

(ii) Hence, by letting $x = \frac{1}{16}$, compute an approximation of $\sqrt[4]{17} + \sqrt[4]{15}$ to four decimal places. **[4 marks]**

(b) Show that the coefficient of the x^5 term of the product $(x + 2)^5 (x - 2)^4$ is 96. **[7 marks]**

(c) (i) Use the Intermediate Value Theorem to prove that $x^3 = 25$ has at least one root in the interval [2, 3]. **[3 marks]**

(ii) The table below shows the results of the first four iterations in the estimation of the root of $f(x) = x^3 - 25 = 0$ using interval bisection.

The procedure used $a = 2$ and $b = 3$ as the starting points and p_n is the estimate of the root for the n^{th} iteration.

n	a_n	b_n	p_n	$f(p_n)$
1	2	3	2.5	–9.375
2	2.5	3	2.75	–4.2031
3	2.75	3	2.875	–1.2363
4	2.875	3	2.9375	0.3474
5				
6				
......				
......				

Complete the table to obtain an approximation of the root of the equation $x^3 = 25$ correct to 2 decimal places. **[7 marks]**

Total 25 marks

SECTION C (Module 3)

Answer BOTH questions.

5. (a) Three letters from the word BRIDGE are selected one after the other without replacement. When a letter is selected, it is classified as either a vowel (V) or a consonant (C).

Use a tree diagram to show the possible outcomes (vowel or consonant) of the THREE selections. Show all probabilities on the diagram. **[7 marks]**

(b) (i) The augmented matrix for a system of three linear equations with variables x, y and z respectively is

$$A = \left(\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ -5 & 1 & 1 & 2 \\ 1 & -5 & 3 & 3 \end{array}\right)$$

By reducing the augmented matrix to echelon form, determine whether or not the system of linear equations is consistent. **[5 marks]**

(ii) The augmented matrix for another system is formed by replacing the THIRD row of A in (i) above with $(1\ -5\ \ 5 \mid 3)$.

Determine whether the solution of the new system is unique. **Give a reason for your answer.** **[5 marks]**

(c) A country, X, has three airports (A, B, C). The percentage of travellers that use each of the airports is 45%, 30% and 25% respectively. Given that a traveller has a weapon in his/her possession, the probability of being caught is, 0.7, 0.9 and 0.85 for airports A, B, and C respectively.

Let the event that:

- the traveller is caught be denoted by D, and
- the event that airport A, B, or C is used be denoted by A, B, and C respectively.

(i) What is the probability that a traveller using an airport in Country X is caught with a weapon? **[5 marks]**

(ii) On a particular day, a traveller was caught carrying a weapon at an airport in Country X. What is the probability that the traveller used airport C? **[3 marks]**

Total 25 marks

6. (a) (i) Obtain the general solution of the differential equation

$$\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x.$$ **[7 marks]**

(ii) Hence, given that $y = \frac{15\sqrt{2}\,\pi^2}{32}$, when $x = \frac{\pi}{4}$, determine the constant of the integration. **[5 marks]**

(b) The general solution of the differential equation

$$y'' + 2y' + 5y = 4 \sin 2t$$

is $y = CF + PI$, where CF is the complementary function and PI is a particular integral.

(i) a) Calculate the roots of

$\lambda^2 + 2\lambda + 5 = 0$, the auxiliary equation. **[2 marks]**

b) Hence, obtain the complementary function (CF), the general solution of

$$y'' + 2y' + 5y = 0.$$ **[3 marks]**

(ii) Given that the form of the particular integral (PI) is

$$u_p(t) = A \cos 2t + B \sin 2t,$$

Show that $A = -\frac{16}{17}$ and $B = \frac{4}{17}$. **[3 marks]**

(iii) Given that $y(0) = 0.04$ and $y'(0) = 0$, obtain the general solution of the differential equation. **[5 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

Caribbean Examinations Council

Advanced Proficiency Examination

Pure Mathematics Unit 2

Specimen Paper 01

1 hour 30 minutes

Read The Following Instructions Carefully

1. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

2. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

<u>Sample Item</u>

If the function f (x) is defined by f (x) = cos $\sqrt{x}$ then $\mathrm{f}'(x)$ is

(A) $\frac{1}{2\sqrt{x}}\sin\sqrt{x}$ (B) $\frac{1}{\sqrt{x}}\sin\sqrt{x}$ (C) $-\frac{1}{2}\sin\sqrt{x}$ (D) $-\frac{1}{2\sqrt{x}}\sin\sqrt{x}$

<u>Sample Answer</u>

(A) (B) (C) (■)

The best answer to this item is "$-\frac{1}{2\sqrt{x}}\sin\sqrt{x}$", so answer (D) has been blackened.

3. If you want to change your answer, be sure to erase your old answer completely and fill in your new choice.

4. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can come back to the harder item later.

5. You may do any rough work in this booklet.

6. The use of non-programmable calculators is allowed.

7. This test consists of 45 items. Each correct answer carries one mark.

Module 1

1. If z and z^* are two conjugate complex numbers, where $z = x + \mathrm{i}y$, $x, y \in \mathbb{R}$, then $z\,z^* =$

(A) $x^2 + y^2$ (B) $x^2 - y^2 - 2xy\mathrm{i}$

(C) $x^2 - y^2$ (D) $x^2 + y^2 - 2xy\mathrm{i}$

2. If $|z + \mathrm{i}| = |z + 1|$, where z is a complex number, then the locus of z is

(A) $y = 0$ (B) $y = x$

(C) $y = 1$ (D) $y = \dfrac{x}{2}$

3. Given that $z + 3z^* = 12 + 8\mathrm{i}$, then $z =$

(A) $-3 - 4\mathrm{i}$ (B) $3 + 4\mathrm{i}$

(C) $3 - 4\mathrm{i}$ (D) $-3 + 4\mathrm{i}$

4. One root of a quadratic equation is $2 - 3\mathrm{i}$. The quadratic equation is

(A) $x^2 + 4x + 13 = 0$ (B) $x^2 - 4x - 13 = 0$

(C) $x^2 + 4x - 13 = 0$ (D) $x^2 - 4x + 13 = 0$

5. If $z = \cos\theta + \mathrm{i}\sin\theta$, then $z^4 + \dfrac{1}{z^4} =$

(A) $\cos 4\theta + \mathrm{i}\sin 4\theta$ (B) $4\cos\theta$ - $\mathrm{i}\,(4\sin\theta)$

(C) $2\cos 4\theta$ (D) $2\mathrm{i}\sin 4\theta$

6. The gradient of the normal to the curve with equation $xy^3 + y^2 + 1 = 0$ at the point (2, -1) is

(A) $-\frac{1}{4}$ (B) 4

(C) $\frac{1}{4}$ (D) -4

7. If $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5x}{y}$, then $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} =$

(A) $\frac{5}{y^3} - \frac{25x^2}{y^2}$ (B) $\frac{5}{y} - \frac{25x^2}{y^3}$

(C) $\frac{1}{y} - \frac{25}{xy}$ (D) $\frac{5}{xy} - \frac{25}{x^3y^3}$

8. The curve C is given by the parametric equations $x = t + \mathrm{e}^{-t}$, $y = 1 - \mathrm{e}^{-t}$. The gradient function for C at the point (x, y) is given as

(A) $\frac{1}{\mathrm{e}^t - 1}$ (B) $\frac{1}{1 - \mathrm{e}^t}$

(C) $\frac{-1}{1 + \mathrm{e}^t}$ (D) $\frac{1}{\mathrm{e}^t + 1}$

9. Given $y = a \arccos(ax)$, where a is a constant, $\frac{\mathrm{d}y}{\mathrm{d}x} =$

(A) $\frac{a^2}{\sqrt{(1 - a^2x^2)}}$ (B) $-\frac{1}{\sqrt{(1 - a^2x^2)}}$

(C) $-\frac{a^2}{\sqrt{(1 - a^2x^2)}}$ (D) $\frac{1}{\sqrt{(1 - a^2x^2)}}$

10. Given that $x^2y + y^2z - z^2x$ then $\dfrac{\partial w}{\partial y} =$

(A) $x^2y + 2yz$ (B) $x^2 + 2yz$

(C) $x^2 + y^2$ (D) $x^2 + y^2 + z^2$

11. $\displaystyle\int \frac{x^3}{(x^2 - 3x + 2)}\,\mathrm{d}x$ may be expressed as

(A) $\displaystyle\int \left(\frac{Px + Q}{x^2 - 3x + 2}\right)\mathrm{d}x$ (B) $\displaystyle\int \left(x + 3 + \frac{Px + Q}{x^2 - 3x + 2}\right)\mathrm{d}x$

(C) $\displaystyle\int \left(\frac{P}{x - 1} + \frac{Q}{x - 2}\right)\mathrm{d}x$ (D) $\displaystyle\int \left(x + 3 + \frac{P}{x - 1} + \frac{Q}{x - 2}\right)\mathrm{d}x$

12. $\displaystyle\int \frac{1}{\sqrt{(a^2 - x^2)}}\,\mathrm{d}x =$

(A) $a\sqrt{(a^2 - x^2)} + C$ (B) $\arcsin(x^2) + C$

(C) $\arcsin\left(\dfrac{x}{a}\right) + C$ (D) $a \arcsin(ax) + C$

13. Given that $y = \dfrac{\pi}{2} - x$, then $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x \,\mathrm{d}x =$

(A) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 y \,\mathrm{d}y$ (B) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 y \,\mathrm{d}y$

(C) $\displaystyle\int_0^{\frac{\pi}{2}} (\sin x + \cos x)\,\mathrm{d}x$ (D) $\displaystyle\int_0^{\frac{\pi}{2}} (\sin y + \cos y)\,\mathrm{d}y$

14. Given $I_n = \int tan^n\, x\, \mathrm{d}x$, for $n > 2$, $I_n =$

(A) $\frac{1}{n-1}\tan^{n-1} x + I_{n-2}$

(B) $\frac{1}{n-1}\tan^{n-1} x \sec^2 x - I_{n-2}$

(C) $\tan^{n-1} x - I_{n-2}$

(D) $\frac{1}{n-1}\tan^{n-1} x - I_{n-2}$

15. The value of

$$\int_0^{\pi/2} x \cos \mathrm{d}x$$

is

(A) $\frac{\pi}{2}$

(B) 1

(C) $\frac{\pi}{2} - 1$

(D) $\frac{\pi}{2} + 1$

<u>Module 2</u>

16. Given that a sequence of positive integers $\{U_n\}$ is defined by $U_1 = 2$ and $U_{n+1} = 3U_n + 2$, then $U_n =$

(A) $3^n - 1$ (B) $3^n + 1$

(C) $3n - 1$ (C) $3n + 2$

17. A sequence $a_n = \frac{3n^2 - n + 4}{2n^2 + 1}$

(A) converges (B) is periodic

(C) is alternating (D) diverges

18. The nth term of a sequence is given by $u_n = 9 - 4\left(\frac{1}{2}\right)^{n-1}$. The 5th term of the sequence is

(A) $\frac{71}{8}$ (B) $\frac{35}{4}$

(C) $\frac{37}{4}$ (D) $\frac{9}{4}$

19. $\sum_{r=1}^{m-1} 3\left(\frac{1}{2}\right)^{m-1} =$

(A) $3 - 3 \times 2^{-m}$ (B) $6 - 3 \times 2^{(m-1)}$

(C) $6 - 3 \times 2^{(1-m)}$ (D) $3 - 3 \times 2^{(1-m)}$

20. Given that $\sum_{r=1}^{n} u_n = 5n + 2n^2$, then $u_n =$

(A) $2n^2 + n - 3$ (B) $4n^2 + 4n + 7$

(C) $5n + 2$ (D) $4n + 3$

21. The sum to infinity, $S(x)$, of the series $1 + \left(\frac{2}{1+x}\right) + \left(\frac{2}{1+x}\right)^2 + \left(\frac{2}{1+x}\right)^3 + \ldots$ is

(A) 1 (B) $\frac{x+2}{x+1}$

(C) $\frac{x+1}{x-1}$ (D) $1 + \left(\frac{2}{x+1}\right)^n$

22. The Maclaurin's series expansion for $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots$ has a general term best defined as

(A) $(-1)^n \frac{x^{n+1}}{(n+1)!}$

(B) $(-1)^{n+1} \frac{x^{n+1}}{(n+1)!}$

(C) $(-1)^n \frac{x^{2n+1}}{(n+1)!}$

(D) $(-1)^{n+1} \frac{x^{2n-1}}{(2n-1)!}$

23. The first 2 non-zero terms of the expansion of $\sin(x + \pi/6)$ are

(A) $\frac{1}{2} + \frac{\sqrt{3}}{x} x$

(B) $\frac{1}{2} - \frac{\sqrt{3}}{2} x$

(C) $\frac{1}{2} + \frac{1}{2} x$

(D) $\frac{\sqrt{3}}{2} + \frac{1}{2} x$

24. If ${}^{n}C_{r} = {}^{n-1}C_{r-1}$, then

(A) $n = r$

(B) $n - r = 1$

(C) $r - n = 1$

(D) $n - 1 = r + 1$

25. If $\left(2x^2 - \frac{2}{x}\right)^6 = \ldots + k + \ldots$, where k is independent of x, then $k =$

(A) -960

(B) -480

(C) 480

(D) 960

26. An investment prospectus offers that for an initial deposit of \$5 000 at January 1st an interest rate of 3% will be applied at December 31st on the opening balance for the year. Assuming that no withdrawals are made for any year, the value of the investment n years after the initial deposit is given by

(A) $(5\,000)(1.03)^n$

(B) $(5\,000)(0.03)^{n-1}$

(C) $(5\,000)(1.03)^{n-1}$

(D) $(5\,000)(0.03)^n$

27. The coefficient of x^3 in the expansion of $(1 + x + x^2)^5$ is

(A) 5 (B) 30

(C) 20 (D) 40

28. The equation $\sin x^2 + 0.5x - 1 = 0$ has a real root in the interval

(A) (0.8, 0.9) (B) (0.7, 0.8)

(C) (0.85, 0.9) (D) (0.9, 0.10)

29. $f(x) = x^3 - \frac{7}{x} + 2$, $x > 0$. Given that $f(x)$ has a real root α in the interval (1.4, 1.5), using the interval bisection once α lies in the interval

(A) (1.45, 1.5) (B) (1.4, 1.45)

(C) (1.425, 1.45) (D) (1.4, 1.425)

30. $f(x) = x^3 - x^2 - 6$. Given that $f(x) = 0$ has a real root α in the interval [2.2, 2.3], applying linear interpolation once on this interval an approximation to α, correct to 3 decimal places, is

(A) 2.217 (B) 2.216

(C) 2.219 (D) 2.218

<u>Module 3</u>

31. Taking 1.6 as a first approximation to α, where the equation $4 \cos x + e^{-x} = 0$ has a real root α in the interval (1.6, 1.7), using the Newton-Raphson method a second approximation to α (correct to 3 decimal places) ≈

(A) 1.620 (B) 1.622

(C) 1.635 (D) 1.602

32. $f(x) = 3x^3 - 2x - 6$. Given that $f(x) = 0$ has a real root, α, between $x = 1.4$ and $x = 1.45$, starting with $x_0 = 1.43$ and using the iteration

$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$, the value of x_1 correct to 4 decimal places is

(A) 1.4370 (B) 1.4372

(C) 1.4371 (D) 1.4369

33. Ten cards, each of a different colour, and consisting of a red card and a blue card, are to be arranged in a line. The number of different arrangements in which the red card is not next to the blue card is

(A) $10! - 2! \times 2!$ (B) $10! - 9! \times 2!$

(C) $8! - 2! \times 2!$ (D) $9! - 2 \times 2!$

34. The number of ways in which all 10 letters of the word **STANISLAUS** can be arranged if the **S**s must all be together is

(A) $\frac{8! \times 3!}{2!}$ (B) $8! \times 3!$

(C) $\frac{8!}{3!}$ (D) $\frac{8!}{2!}$

35. A committee of 4 is to be chosen from 4 teachers and 4 students. The number of different committees that can be chosen if there must be at least 2 teachers is

(A) 53 (B) 192

(C) 36 (D) 45

36. A and B are two events such that P (A) = p and P (B) = $\frac{1}{3}$. The probability that neither occurs is $\frac{1}{2}$. If A and B are mutually exclusive events then p =

(A) $\frac{5}{6}$ (B) $\frac{2}{3}$

(C) $\frac{1}{5}$ (D) $\frac{1}{6}$

37. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively. The probability that on a randomly chosen day Bill travels by foot and is late is

(A) $\frac{3}{10}$ (B) $\frac{13}{30}$

(C) $\frac{1}{30}$ (D) $\frac{1}{10}$

38. Given $\begin{vmatrix} 6 & 0 & 1 \\ 7 & 7 & 0 \\ 0 & -12 & x \end{vmatrix} = 0,$

the value of x is

(A) -2 (B) 2

(C) 12 (D) 7

Items 39 – 40 are based on the matrix $A = \begin{pmatrix} 2 & -7 & 8 \\ 3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$

39. The transpose of matrix A results in $|A|$ being

(A) negative

(B) squared

(C) 0

(D) unchanged

40. The matrix resulting from adding Row 1 to –Row 2 is

(A) $\begin{pmatrix} -1 & -1 & 13 \\ 3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & -7 & 8 \\ -1 & -1 & -13 \\ 4 & 0 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} -5 & 7 & 8 \\ -3 & -6 & -5 \\ 4 & 0 & -1 \end{pmatrix}$

(D) $\begin{pmatrix} -5 & 5 & -8 \\ 3 & -3 & 5 \\ 4 & -0 & 1 \end{pmatrix}$

41. Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}$ $B = \begin{pmatrix} -4 & 0 & 2 \\ 0 & 6 & -4 \\ 2 & -4 & 2 \end{pmatrix}$, by considering AB, then $A^{-1} =$

(A) $2B$

(B) B

(C) $\frac{1}{2}B$

(D $\frac{1}{2}AB$

42. The general solution of the differential equation

$$\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x \text{ is found by evaluating}$$

(A) $\int \frac{d}{dx} y \sin x \, dx = \int 2 \cos x \, dx$

(B) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int 2 \cos x \, dx$

(C) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \sin 2x \, dx$

(D) $\int \frac{d}{dx} \frac{y}{\sin x} \, dx = \int \cos x \, dx$

43. The general solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 3e^x$ is of the form

(A) $y = Ae^x + Be^{2x} + ke^x$

(B) $y = Ae^x + Be^{2x} - 3e^x$

(C) $y = Ae^{-x} + Be^{-2x} + kxe^x$

(D) $y = Ae^x + Be^{2x} + kxe^x$

44. A particular integral of the differential equation

$$\frac{d^2y}{dx^2} + 25y = 3 \cos 5x$$

is of the form $y = \lambda x \sin 5x$. The general solution of the differential equation is

(A) $y = A \cos 5x - B \sin 5x - \lambda x \sin 5x$

(B) $y = A \cos 5x + B \sin 5x + \lambda x \sin 5x$

(C) $y = A \cos 5x + B \sin 5x - \lambda x \sin 5x$

(D) $y = A \cos 5x - B \sin 5x + \lambda x \sin 5x$

45. The general solution of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2x^2 + x - 1$$

is

(A) $y = e^{2x}(A + Bx) + ax^2 + bx + c$

(B) $y = e^{-2x}(A + Bx) + ax^2 + bx + c$

(C) $y = e^{2x}(A + Bx) + 2x^2 + x - 1$

(D) $y = e^{2x}(A - Bx) + ax^2 + bx + c$

End of Test

Key

Unit 2 Paper 01

Module	Item	Key	S. O.	Module	Item	Key	S. O.
1	1	A	A2	3	31	A	D5
	2	B	A11		32	C	D6
	3	C	A5		33	B	A2
	4	D	A3		34	D	A3
	5	C	A12		35	A	A4
	6	D	B4		36	D	A14
	7	B	B9		37	C	A16
	8	A	B3		38	B	B2
	9	C	B8		39	D	B1
	10	B	B6		40	A	B3
	11	D	C3		41	C	B7
	12	C	C9		42	B	C1
	13	A	C7		43	D	C3 (ii) (i)
	14	D	C10		44	B	C3 (iii) (iii)
	15	C	C8		45	A	C3 (i) (ii)
2	16	A	A1				
	17	A	A3				
	18	B	A2				
	19	C	B4				
	20	D	B3				
	21	C	B6				
	22	D	B8				
	23	A	B9				
	24	A	C1				
	25	D	C3				
	26	C	C4				
	27	B	C3				
	28	A	D1				
	29	B	D2				
	30	D	D3				

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CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

SPECIMEN PAPER

UNIT 2

COMPLEX NUMBERS, ANALYSIS AND MATRICES

PAPER 02

SOLUTIONS AND MARK SCHEMES

SECTION A

(MODULE 1)

Question 1

(a) (i) $\frac{4-2\text{i}}{1-3\text{i}} = \frac{(4-2\text{i})(1+3\text{i})}{1-3\text{i})(1+3\text{i})}$

$= \frac{4+12\text{i}-2\text{i}-6\text{i}^2}{1+9}$ (1 mark)

$= \frac{4+10\text{i}+6}{10}$ (1 mark)

$= \frac{10+10\text{i}}{10}$ (1 mark)

$= 1+\text{i}$ (1 mark)

[4 marks]

(ii) arg is $\tan^{-1}(1)\,) = \frac{\pi}{4}$ (1 mark)

[1 mark]

(b) (i) Let $u = x + \text{i}\,y$, where x, y are real nos.

$u^2 = -5 + 12\text{i}$ □ $(x + \text{i}y)^2 = -5 + 12\text{i}$

□ $x^2 - y^2 + 2\text{i}xy = -5 + 12\text{i}$ (1 mark)

□ $x^2 - y^2 = -5, \quad 2xy = 12$ (1 mark)

□ $x^2 - \left(\frac{6}{x}\right)^2 = -5, \quad y = \frac{6}{x}$

□ $x^2 - \frac{36}{x^2} = -5$ (1 mark)

□ $(x^2)^2 + 5x^2 - 36 = 0$ (1 mark)

□ $(x^2 + 9)(x^2 - 4) = 0$ (1 mark)

□ $x^2 = -9(\text{inadmissable}), x^2 = 4$ (1 mark)

□ $x = \pm 2, y =$ □3 (1 mark)

□$= 2 - 3\text{i}$ or $- 2 + 3\text{i}$ (1 mark)

[8 marks]

(b) (ii) $z^2 + \mathrm{i}z + (1 - 3\mathrm{i}) = 0$ □ $z = \frac{-\mathrm{i} \pm \sqrt{\mathrm{i}^2 - 4(1-3\mathrm{i})}}{2}$ (1 mark)

□ $z \frac{-\mathrm{i} \pm \sqrt{-1-4+12\mathrm{i}}}{2}$ (1 mark)

□ $z \frac{-\mathrm{i} \pm \sqrt{-5+12\mathrm{i}}}{2}$ (1 mark)

□ $z \frac{-\mathrm{i} \pm 2-3\mathrm{i}}{2}$ (1 mark)

□ $z \frac{2-4\mathrm{i}}{2}$ or $\frac{2+2\mathrm{i}}{2}$ (1 mark)

□ $z = 1 - 2\mathrm{i}$ or $-1 + \mathrm{i}$ (1 mark)

[6 marks]

(c) $(1 + 3\mathrm{i})z + (4 - 2\mathrm{i})z = 10 + 4\mathrm{i}$, and $z = a + \mathrm{i}b$

□ $(1 + 3\mathrm{i})(a + \mathrm{i}b) + (4 - 2\mathrm{i})(a - \mathrm{i}b) = 10 + 4\mathrm{i}$ (1 mark)

□ $(a - 3b) + \mathrm{i}(3a + b) + (4a - 2b) + \mathrm{i}(-4b - 2a) = 10 + 4\mathrm{i}$ (1 mark)

□ $a - 3b + 4a - 2b = 10$ and $3a + b - 4b - 2a = 4$ (1 mark)

□ $5a - 5b = 10$ and $a - 3b = 4$ (1 mark)

□ $a = 1, b = -1$ (1 mark)

□ $z = 1 - \mathrm{i}$ (1 mark)

[6 marks]

Total 25 marks

Specific Objectives (A) 1, 4, 5, 6, 7, 8.

Question 2

(a) Let I = $\int e^{3x} \sin 2x \, dx$

$= \frac{1}{3} e^{3x} \sin 2x - \int \frac{e^{3x}}{3} (2\cos 2x) \, dx$ (2 marks)

$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \int e^{3x} (2\cos 2x) \, dx$

$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{3} \left| \frac{1}{3} e^{3x} \cos 2x + \int \frac{e^{3x}}{3} (2\sin 2x) \, dx \right|$ (2 marks)

$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x - \frac{4}{9} \int e^{3x} \sin 2x dx$

$= \frac{1}{3} e^{3x} \sin 2x - \frac{2}{9} e^{3x} \cos 2x - \frac{4}{9} I$ (1 mark)

□ $I + \frac{4}{9} I = \frac{1}{9} (3 \sin 2x - 2\cos 2x)$ (1 mark)

□ $I = \frac{1}{13} (3 \sin 2x - 2\cos 2x)$ + constant (1 mark)

[7 marks]

Alternatively

$\int e^{3x} e^{2ix} dx = \int e^{(3+2i)x} \, dx$ (2 marks)

□ Im $\left| \frac{e^{(3+2i)x}}{3+2i} \right|$ + constant (2 marks)

□ $\int e^{3x} \sin 2x dx = \text{Im} \frac{(3-2i)}{13} e^{3x} (\cos 2x + i\sin 2x)$ (2 marks)

$\frac{e^{3x}}{13} (3 \sin 2x - 2\cos 2x)$ + const. (1 mark)

[7 marks]

(b) (i) a) $y = \tan^{-1}(3x)$ □ $\tan y = 3x$ (1 mark)

□ $\sec^2 y \frac{dy}{dx} = 3$

□ $\frac{dy}{dx} = \frac{3}{\sec^2 y}$ (1 mark)

□ $\frac{dy}{dx} = \frac{3}{1+\tan^2 y}$ (1 mark)

□ $\frac{dy}{dx} = \frac{3}{1 + 9x^2}$ (1 mark)

[4 marks]

b) $\int \frac{x+2}{1 + 9x^2} dx = \int \frac{x}{1 + 9x^2} \, dx + 2 \int \frac{1}{1 + 9x^2} \, dx$ (1 mark)

$= \frac{1}{18} \ln (1 + 9x^2) + \frac{2}{3} \tan^{-1}(3x)$ + constant (3 marks)

[4 marks]

(b) (ii) $y = \frac{\ln(5x)}{x^2}$, Using the product rule: $y = \frac{1}{x^2}\ln(5x)$ (1 mark)

$$□ \frac{dy}{dx} = -\frac{2}{x^3}\ln(5x) + \frac{1}{x^2} \times \frac{1}{x}$$ (2 marks)

$$= \frac{1-2\ln(5x)}{x^3}$$ (1 mark)

$$= \frac{1-\ln(25x^2)}{x^3}$$ (1 mark)

[5 marks]

c) $f(x, y) = x^2 + y^2 - 2xy$

(i) $\frac{\partial f}{\partial x} = 2x - 2y$ $\frac{\partial f}{\partial y} = 2y - 2x$ [2 marks]

(ii) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x(2x - 2y) + y(2y - 2x)$

$= 2x^2 + 2y^2 - 4xy$ (1 mark)

$= 2(x^2 + y^2 - 2xy)$ (1 mark)

$= 2(f(x, y)$ (1 mark)

[3 marks]

Total 25 marks

Specific Objectives: (A) 13, (B) 1, 2, 5, 6, 8, (C) 4, 5, 6, 8

SECTION B

(MODULE 2)

Question 3

(a) (i) $$\frac{1}{(2r-1)(2r+1)} \equiv \frac{A}{2r-1} + \frac{B}{2r+1}$$

$\Rightarrow \quad 1 = A(2r+1) + B(2r-1)$ (1 mark)

$\Rightarrow \quad 0 = 2A + 2B$ and $A - B = 1$ (2 marks)

$\Rightarrow \quad A = \frac{1}{2}$ and $B = -\frac{1}{2}$ (2 marks)

[5 marks]

(ii) $$S = \sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \sum_{r=1}^{n} \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$$ (1 mark)

$$= \frac{1}{2}\left(\frac{1}{1} - \frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) + \frac{1}{2}\left(\frac{1}{5} - \frac{1}{7}\right) + \ldots + \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

(3 marks)

$$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$ (1 mark)

[5 marks]

(iii) As $n \to \infty$, $\frac{1}{2n+1} \to 0$ (2 marks)

Hence $S_\infty = \frac{1}{2}$ (1 mark)

[3 marks]

(b) (i) $S = 1(2) + 2(5) + 3(8) + \ldots$

In each term, 1st factor is in the natural sequence and the second factor differs by 3 (1 mark)

$\Rightarrow$ the r^{th} term is $r(3r - 1)$ (1 mark)

[2 marks]

(ii) $$S_n = \sum_{r=1}^{n} r(3r-1)$$

for $n = 1$ $\quad S_1 = \sum_{r=1}^{1} r(3r-1) = 1 \times 2 = 2$

and $\quad 1^2\,(1+1) = 1 \times 2 = 2$ (1 mark)

hence, $\quad S_n = n^2\,(n+1)$ is true for $n = 1$ (1 mark)

Assume $\quad S_n = n^2\,(n+1)$ for $n = k \in \mathbb{N}$ (1 mark)

that is, $\quad S_k = k^2\,(k+1)$ (1 mark)

Then, $S_{k+1} = \sum_{r=1}^{k+1} r\left(3r-1\right) = S_k + \left(k+1\right)\left(3k+2\right)$ (1 mark)

$= k^2\left(k+1\right) + \left(k+1\right)\left(3k+2\right)$ (1 mark)

$= \left(k+1\right)\left[k^2 + 3k + 2\right]$ (1 mark)

$\Rightarrow \quad S_{k+1} = \left(k+1\right)\left[\left(k+1\right)\left(k+2\right)\right]$

$= \left(k+1\right)^2 \left[\left(k+1\right)+1\right]$ (1 mark)

$\Rightarrow$ true for $n = k + 1$ whenever it is assumed true for $n = k$, (1 mark)

$\Rightarrow$ true for all $n \in N$

$\Rightarrow \quad S_n = n^2\left(n+1\right)\ n$ □ $\mathbb{N}$. (1 mark)

[10 marks]

Total 25 marks

Specific Objectives: (B) 1, 3, 5, 6, 10

Question 4

(a) (i) Let $S \equiv \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^{10}} + \ldots$

$$\frac{\frac{1}{2^4}}{\frac{1}{2}} = \frac{\frac{1}{2^7}}{\frac{1}{2^4}}$$ (1 mark)

$$= \frac{1}{2^3}$$ (1 mark)

$\therefore$ S is geometric with common ratio $r = \frac{1}{2^3}$ (1 mark)

[3 marks]

(ii) $$S_n = \frac{\frac{1}{2}\left[1 - \left(\frac{1}{2}\right)^{3n}\right]}{1 - \left(\frac{1}{2}\right)^3}$$ (1 mark)

$$= \frac{\frac{1}{2}\left[1 - \frac{1}{2^{3n}}\right]}{1 - \frac{1}{8}}$$ (1 mark)

$$= \frac{1}{2} \times \frac{8}{7}\left[1 - \frac{1}{2^{3n}}\right]$$ (1 mark)

$$= \frac{4}{7}\left[1 - \frac{1}{2^{3n}}\right]$$ (1 mark)

[4 marks]

(b) (i) $f(x) = \cos 2x \quad \Rightarrow \quad f^{1}(x) = -2\sin\ 2x$ (1 mark)

$\Rightarrow \quad f^{11}(x) = -4\cos 2x$ (1 mark)

$\Rightarrow \quad f^{111}(x) = 8\ \sin 2x$ (1 mark)

$\Rightarrow \quad f^{iv}(x) = 16\ \cos 2x$ (1 mark)

so, $f(0) = 1,\ \ f^{1}(0) = 0,\ \ f^{11}(x) = -4,\ \ f^{111}(0) = 0,\ \ f^{iv}(0) = 16$ (1 mark)

Hence, by Maclaurin's Theorem,

$$\cos 2x \ = \ 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \qquad \text{(1 mark)}$$

$$= 1 - 2x^2 + \frac{2}{3}x^4 \qquad \text{(1 mark)}$$

[7 marks]

(c) (i) $\sqrt{\left(\frac{1+x}{1-x}\right)}$

$= (1+x)^{½}\ (1-x)^{-½}$ (1 mark)

$$= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \ldots\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 \ldots\right) \qquad \text{(3 marks)}$$

$$= \ 1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 \qquad \text{(2 marks)}$$

for $-1 < x < 1$ (1 mark)

[7 marks]

(ii) $\sqrt{\frac{1.02}{0.98}} = \sqrt{\frac{102}{98}} = \frac{1}{7}\sqrt{51}$ (1 mark)

$\sqrt{51} \ = \ 7\sqrt{\frac{1+x}{1-x}}$ where $x = 0.02$ (1 mark)

$$\Rightarrow \sqrt{51} \ = \ 7\left\{1 + 0.02 + \frac{1}{2}(0.02)^2 + \frac{1}{2}(0.02)^3\right\} \qquad \text{(1 mark)}$$

$= 7.14141$ (5 d.p.) (1 mark)

[4 marks]

Specific Objectives: (B) 5, 9, 11; (C)3, 4

Total 25 marks

SECTION C

(MODULE 3)

Question 5

(a) (i) P (First card drawn has even number) $= \frac{5}{10} = 1/2$ (1 mark)

P (Second card drawn has even number) $= \frac{4}{9}$ (2 marks)

$\therefore$ P (Both cards have even numbers) $= \left(\frac{1}{2}\right)\left(\frac{4}{9}\right)$

$= \frac{2}{9}$ (1 mark)

[4 marks]

(ii) P (Both cards have odd numbers) $= \frac{2}{9}$ (1 mark)

P (One card has odd and the other has even i.e. both cards do not have odd or do not have even numbers) $= 1 - 2\left(\frac{2}{9}\right)$ (2 marks)

$= \frac{5}{9}$ (1 mark)

[3 marks]

(b) (i) a) $\frac{82}{150} = 0.547$ [2 marks]

ii) $\frac{39}{150} + \frac{75}{150} = 0.76$ [4 marks]

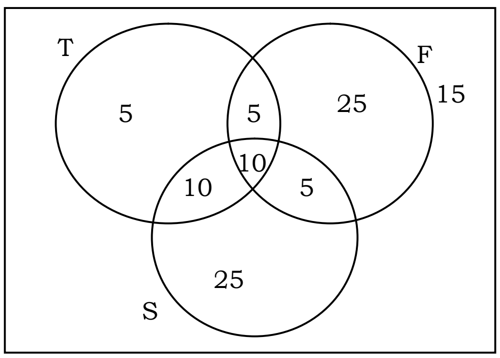
iii) $\frac{82}{150} + \frac{39}{150} - \frac{27}{150} = 0.267$ [3 marks]

(c) Let T, S and F represent respectively the customers purchasing tools, seeds and fertilizer.

(i) One mark for any two correct numbers (4 marks)

(ii) a) 5 (1 mark)
b) 10 (1 mark)
c) 5 (1 mark)
d) 15 (1 mark)

(i)



Venn diagram

Total 25 marks

Specific Objectives: (A) 5, 6, 7, 9, 10, 11, 12, 13

Question 6

(a) $\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$

$5\,(2x - 2) - x\,(x^2 + 2x + 3) + 3\,\,(2x + 4 + 6) = 0$ (3 marks)

$x^3 + 2x^2 - 13x - 20 = 0$ (1 mark)

Subs $x = -4$, $(-4)^3 + 2\,(-4)^2 - 13(-4) - 20 = 0$

$(x+4)\,(x^2 - 2x - 5) = 0$ (2 marks)

$x = -4$

$x = \dfrac{2 \pm \sqrt{24}}{2}$

$x = 1 \pm \sqrt{6}$ (2 marks)

[8 marks]

Alternatively

$\begin{vmatrix} 5 & x & 3 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$ □ $\begin{vmatrix} 4+x & x+4 & x+4 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$ (Add rows 2 and 3 to row 1)

□$(x+4)\begin{vmatrix} 1 & 1 & 1 \\ x+2 & 2 & 1 \\ -3 & 2 & x \end{vmatrix} = 0$ □ $(x+4)\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & -1 \\ -5 & 2 & x-2 \end{vmatrix} = 0$

(subtract columns 2 from Columns 1 and 3).

□ $(x+4)x - (x^2 - 2x - 5) = 0$ □ $x = -4$ or $1 \pm \sqrt{6}$ [8 marks]

(b) $y \tan x \,\frac{\mathrm{d}y}{\mathrm{d}x} = (4 + y^2)\sec^2 x$ (1 mark)

$\dfrac{y}{4+y^2}\,\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sec^2 x}{\tan x}$

$\dfrac{y\,\mathrm{d}y}{4+y^2} = \dfrac{\sec^2 x\,\mathrm{d}x}{\tan x}$

$\int \dfrac{y\,\mathrm{d}y}{4+y^2} = \int \dfrac{\sec^2 x\,\mathrm{d}x}{\tan x}$ (1 mark)

$\frac{1}{2}\ln(4 + y^2) = \ln|\tan x| + c$ (2 marks)

[4 marks]

(c) (i) $y = u\cos 3x + v\sin 3x$

$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -3u\sin 3x + 3v\cos 3x$ (2 marks)

$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -9u\cos 3x - 9v\sin 3x$ (2 marks)

so, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = -30\sin 3x$

$\Rightarrow -(6v - 12u)\sin 3x + (-6u + 12v)\cos 3x = -30\sin 3x$ (1 mark)

$\Rightarrow 2u + v = 5$ and $u = 2v$ (1 mark)

$\Rightarrow u = 2$ and $v = 1$ (2 marks)

[8 marks]

(ii) the auxiliary equation of the different equation is

$k^2 + 4k + 3 = 0$ (1 mark)

$\Rightarrow (k + 3)(k + 1) = 0$

$\Rightarrow k = -3$ or -1 (2 marks)

$\Rightarrow$ the complementary function is

$y = A\mathrm{e}^{-x} + B\mathrm{e}^{-3x}$; where A, B are constants (1 mark)

General solution is $y = A\mathrm{e}^{-x} + B\mathrm{e}^{-3x} + \sin 3x + 2\cos 3x$ (1 mark)

[5 marks]

Total 25 marks

Specific Objectives: (B) 1, 5, 6; (C) 1, 3

End of Test

FORM TP 02134032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 2

ANALYSIS, MATRICES AND COMPLEX NUMBERS

SPECIMEN PAPER

PAPER 03/B

1 *hour* 30 *minutes*

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable electronic calculator

SECTION A (MODULE 1)

Answer this question.

1. (a) $z = 8 + (8\sqrt{3})\text{i}$.

(i) Find the modulus and argument of z. **[3 marks]**

(ii) Using de Moivre's theorem show that z^3 is real, stating the value of z^3.

[2 marks]

(b) A complex number is represented by the point P in the Argand diagram.

(i) Given that $|z - 6| = |z|$ show that the locus of P is $x = 3$. **[2 marks]**

(ii) Find the complex numbers z which satisfy both

$|z - 6| = |z|$ and $|z - 3 - 4\text{i}| = 5$. **[5 marks]**

(c) Given $I_n = \int_0^8 x^n\,(8 - x)^{1/x}\,\text{d}x, n \geq 0,$ show that

$$I_n = \frac{24n}{3n + 4} I_{n-1}, n \geq 1.$$ **[8 marks]**

Total 20 marks

SECTION B (MODULE 2)

Answer this question

2. (a) (i) Show that $(r + 1)^3 - (r - 1)^3 = 6r^2 + 2$. **[2 marks]**

(ii) Hence show that $\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$. **[5 marks]**

(iii) Show that $\sum_{r=n}^{2n} r^2 = \frac{n}{6}(n+1)(an+b)$, where a and b are constants to be found.

[4 marks]

(b) The displacement x metres of a particle at time t seconds is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + x + \cos x = 0.$$

When $t = 0$, $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0.5$.

Find a Taylor series solution for x in ascending powers of t, up to and including the term in t^3.

[5 marks]

(c) Given that α is the only real root of the equation

$$x^3 - x^2 - 6 = 0,$$

(i) Show that $2.2 < \alpha < 2.3$. **[2 marks]**

(ii) Use linear interpolation once on the interval [2.2, 2.3] to find another approximation to α, giving your answer to 3 decimal places.

[2 marks]

Total 20 marks

GO ON TO NEXT PAGE

SECTION C (MODULE 3)

Answer this question

3. (a) Three identical cans of cola, two identical cans of green tea and two identical cans of orange juice are arranged in a row.

Calculate the number of arrangements if the first and last cans in the row are of the same type of drink.

[3 marks]

(b) Kris takes her dog for a walk every dy. The probability that they go to the park on any day is 0.6. If they go to the park there is a probability of 0.35 that the dog will bark. If they do not go to the park there is probability of 0.75 that the dog will bark.

Find the probability that the dog barks on any particular day. **[2 marks]**

(c) A committee of six people, which must consist of at least 4 men and at least one woman, is to be chosen from 10 men and 9 women.

Find the number of possible committees that include either Albert or Tracey but not both.

[3 marks]

(d) $A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} -3 & 4 & -7 \\ -2 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}$

(i) Find AB. **[2 marks]**

(ii) Deduce A^{-1}. **[2 marks]**

(iii) Given that $B^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 4 \\ 0 & 1 & 1 \end{pmatrix}$, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

[2 marks]

(e) Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\cot x = \sin x.$$

[6 marks]

Total 20 marks

END OF TEST

TEST CODE **02234020**

FORM TP 2014243

MAY/JUNE 2014

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

28 MAY 2014 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of TWO questions.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Differentiate, with respect to x,

$$y = \ln (x^2 + 4) - x \tan^{-1} (\frac{x}{2}).$$ **[5 marks]**

(ii) A curve is defined parametrically as

$$x = a \cos^3 t, y = a \sin^3 t.$$

Show that the tangent at the point $P (x, y)$ is the line

$$y \cos t + x \sin t = a \sin t \cos t.$$ **[7 marks]**

(b) Let the roots of the quadratic equation $x^2 + 3x + 9 = 0$ be α and β.

(i) Determine the nature of the roots of the equation. **[2 marks]**

(ii) Express α and β in the form $re^{i\theta}$, where r is the modulus and θ is the argument, where $-\pi < \theta \leq \pi$. **[4 marks]**

(iii) Using de Moivre's theorem, or otherwise, compute $\alpha^3 + \beta^3$. **[4 marks]**

(iv) Hence, or otherwise, obtain the quadratic equation whose roots are α^3 and β^3. **[3 marks]**

Total 25 marks

2. (a) Let $F_n(x) = \int (\ln x)^n \, dx$.

(i) Show that $F_n(x) = x(\ln x)^n - n F_{n-1}(x)$. **[3 marks]**

(ii) Hence, or otherwise, show that

$$F_3(2) - F_3(1) = 2(\ln 2)^3 - 6(\ln 2)^2 + 12 \ln 2 - 6.$$ **[7 marks]**

(b) (i) By decomposing $\dfrac{y^2 + 2y + 1}{y^4 + 2y^2 + 1}$ into partial fractions, show that

$$\frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1} = \frac{1}{y^2 + 1} + \frac{2y}{(y^2 + 1)^2}.$$ **[7 marks]**

(ii) Hence, find $\displaystyle\int_0^1 \frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1} \, dy$. **[8 marks]**

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Prove, by mathematical induction, that for $n \in \mathrm{N}$

$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}.$$ **[8 marks]**

(ii) Hence, or otherwise, find $\lim_{n \to \infty} S_n$. **[3 marks]**

(b) Find the Maclaurin expansion for

$$f(x) = (1 + x)^2 \sin x$$

up to and including the term in x^3. **[14 marks]**

Total 25 marks

4. (a) (i) For the binomial expansion of $(2x + 3)^{20}$, show that the ratio of the term in x^6 to the term in x^7 is $\frac{3}{4x}$. **[5 marks]**

(ii) a) Determine the FIRST THREE terms of the binomial expansion of $(1 + 2x)^{10}$.

b) Hence, obtain an estimate for $(1.01)^{10}$. **[7 marks]**

(b) Show that $$\frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!} = \frac{(n+1)!}{(n-r+1)!r!}.$$ **[6 marks]**

(c) (i) Show that the function $f(x) = -x^3 + 3x + 4$ has a root in the interval $[1, 3]$. **[3 marks]**

(ii) By taking $x_1 = 2.1$ as a first approximation of the root in the interval $[1, 3]$, use the Newton–Raphson method to obtain a **second** approximation, x_2, in the interval $[1, 3]$. **[4 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C

Module 3

Answer BOTH questions.

5. (a) (i) Five teams are to meet at a round table. Each team consists of two members AND one leader. How many seating arrangements are possible if each team sits together with the leader of the team in the middle? **[7 marks]**

(ii) In an experiment, individuals were asked to colour a shape by selecting from two available colours, red and blue. The individuals chose one colour, two colours or no colour.

In total, 80% of the individuals used colours and 600 individuals used no colour.

a) Given that 40% of the individuals used red and 50% used blue, calculate the probability that an individual used BOTH colours. **[4 marks]**

b) Determine the TOTAL number of individuals that participated in the experiment. **[2 marks]**

(b) **A** and **B** are the two matrices given below.

$$\mathbf{A} = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix}$$

(i) Determine the range of values of x for which $\mathbf{A}^{-1}$ exists. **[4 marks]**

(ii) Given that det $(\mathbf{AB}) = -21$, show that $x = 3$. **[4 marks]**

(iii) Hence, obtain $\mathbf{A}^{-1}$. **[4 marks]**

Total 25 marks

GO ON TO THE NEXT PAGE

6. (a) (i) Show that the general solution of the differential equation

$$y' + y \tan x = \sec x$$

is $y = \sin x + C \cos x$. **[10 marks]**

(ii) Hence, obtain the particular solution where $y = \dfrac{2}{\sqrt{2}}$ and $x = \dfrac{\pi}{4}$. **[4 marks]**

(b) A differential equation is given as $y'' - 5y' = xe^{5x}$. Given that a particular solution is $y_p(x) = Ax^2 e^{5x} + Bxe^{5x}$, solve the differential equation. **[11 marks]**

Total 25 marks

END OF TEST

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TEST CODE **02234020**

FORM TP 2015268

MAY/JUNE 2015

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

27 MAY 2015 (p.m.)

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SECTION A

Module 1

Answer BOTH questions.

1. (a) Three complex numbers are given as

$$z_1 = 1 + (7 - 4\sqrt{3})i, \quad z_2 = \sqrt{3} + 3i \quad \text{and} \quad z_3 = -2 + 2i.$$

(i) Express the quotient $\frac{z_3}{z_2}$ in the form $x + iy$ where $x, y \in \mathrm{R}$. **[3 marks]**

(ii) Given that $\arg w = \arg z_3 - [\arg z_1 + \arg z_2]$, $|z_1| = 1$ and $\arg z_1 = \frac{\pi}{12}$ rewrite

$$w = \frac{z_3}{z_1 z_2} \text{ in the form } re^{i\theta} \text{ where } r = |w| \text{ and } \theta = \arg w.$$ **[6 marks]**

(b) A complex number $v = x + iy$ is such that $v^2 = 2 + i$. Show that

$$x^2 = \frac{2 + \sqrt{5}}{2}.$$ **[7 marks]**

(c) The function f is defined by the parametric equations

$$x = \frac{e^{-t}}{\sqrt{1 - t^2}} \quad \text{and} \quad y = \sin^{-1} t \quad \text{for } -1 < t \leq 0.5.$$

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{e^t(1 - t^2)}{t^2 + t - 1}$. **[6 marks]**

(ii) Hence, show that f has no stationary value. **[3 marks]**

Total 25 marks

2. (a) Let $4x^2 + 3xy^2 + 7x + 3y = 0$.

(i) Use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{8x + 3y^2 + 7}{3(1 + 2xy)}.$$ **[5 marks]**

(ii) Show that for $f(x, y) = 4x^2 + 3xy^2 + 7x + 3y$

$$6\,\frac{\partial f(x, y)}{\partial y} - 10 = \left(\frac{\partial^2 f(x, y)}{\partial y^2}\right)\left(\frac{\partial^2 f(x, y)}{\partial y\,\partial x}\right) + \frac{\partial^2 f(x, y)}{\partial x^2}.$$ **[5 marks]**

(b) The rational function

$$f(x) = \frac{18x^2 + 13}{9x^2 + 4}$$

is defined on the domain $-2 \le x \le 2$.

(i) Express $f(x)$ in the form $a + \dfrac{b}{9x^2 + 4}$ where $a, b \in \mathrm{R}$. **[2 marks]**

(ii) Given that $f(x)$ is symmetric about the y-axis, evaluate $\int_{-2}^{2} f(x)\,dx$. **[6 marks]**

(c) Let h be a function of x.

(i) Show that

$$\int h^n \ln h\, dh = \frac{h^{n+1}}{(n+1)^2}\,[-1 + (n + 1)\ln h] + C,$$

where $-1 \neq n \in \mathrm{Z}$ and $C \in \mathrm{R}$. **[5 marks]**

(ii) Hence, find

$$\int \sin^2 x \cos x \ln(\sin x)\, dx.$$ **[2 marks]**

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) The nth term of a sequence is given by

$$T_n = \frac{2n+1}{\sqrt{n^2+1}}.$$

(i) Determine $\lim_{n \to \infty} T_n$. **[5 marks]**

(ii) Show that $T_4 = \frac{9}{4}\left(1 + \frac{1}{16}\right)^{-\frac{1}{2}}$. **[3 marks]**

(iii) Hence, use the binomial expansion with $x = \frac{1}{16}$ to approximate the value of T_4 for terms up to and including x^3. Give your answer correct to two decimal places. **[4 marks]**

(b) A series is given as

$$2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \ldots$$

(i) Express the n^{th} partial sum S_n of the series in sigma notation. **[2 marks]**

(ii) Hence, given that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges to $\frac{\pi^2}{6}$, show that S_n diverges as $n \to \infty$. **[4 marks]**

(c) Use the method of induction to prove that

$$\sum_{r=1}^{n} r(r-1) = \frac{n(n^2-1)}{3}.$$ **[7 marks]**

Total 25 marks

4. (a) A function is defined as $g(x) = e^{3x+1}$.

(i) Obtain the Maclaurin series expansion for $g(x)$ up to and including the term in x^4. **[6 marks]**

(ii) Hence, estimate $g(0.2)$ correct to three decimal places. **[3 marks]**

(b) (i) Let $f(x) = x - 3\sin x - 1$.

Use the intermediate value theorem to show that f has at least one root in the interval $[-2, 0]$. **[3 marks]**

(ii) Use at least three iterations of the method of interval bisection to show that

$f(-0.538) \approx 0$ in the interval $[-0.7, -0.3]$.

(≈ 0 means approximately equal to 0) **[8 marks]**

(c) Use the Newton–Raphson method with initial estimate $x_1 = 5.5$ to approximate the root of $g(x) = \sin 3x$ in the interval $[5, 6]$, correct to two decimal places. **[5 marks]**

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) Ten students from across CARICOM applied for mathematics scholarships. Three of the applicants are females and the remaining seven are males. The scholarships are awarded to four successful students. Determine the number of possible ways in which a group of FOUR applicants may be selected if

(i) no restrictions are applied **[1 mark]**

(ii) at least one of the successful applicants must be female. **[3 marks]**

(b) Numbers are formed using the digits 1, 2, 3, 4 and 5 without repeating any digit. Determine

(i) the greatest possible amount of numbers that may be formed **[4 marks]**

(ii) the probability that a number formed is greater than 100. **[3 marks]**

(c) A system of equations is given as

$$\begin{aligned} 2x + 3y - z &= -3.5 \\ x - y + 2z &= 7 \\ 1.5x + 3z &= 9 \end{aligned}$$

(i) Rewrite the system of equations as an augmented matrix. **[2 marks]**

(ii) Use elementary row operations to reduce the system to echelon form. **[5 marks]**

(iii) Hence, solve the system of equations. **[3 marks]**

(iv) Show that the system has no solution if the third equation is changed to

$$1.5x - 1.5y + 3z = 9.$$ **[4 marks]**

Total 25 marks

6. (a) Alicia's chance of getting to school depends on the weather. The weather can be either rainy or sunny. If it is a rainy day, the probability that she gets to school is 0.7. In addition, she goes to school on 99% of the sunny school days. It is also known that 32% of all school days are rainy.

(i) Construct a tree diagram to show the probabilities that Alicia arrives at school. **[3 marks]**

(ii) What is the probability that Alicia is at school on any given school day? **[3 marks]**

(iii) Given that Alicia is at school today, determine the probability that it is a rainy day. **[4 marks]**

(b) (i) Show that the equation $y + xy + x^2 = 0$ is a solution of the differential equation

$$\frac{dy}{dx} = \frac{y - x^2}{x(1 + x)}.$$

[5 marks]

(ii) A differential equation is given as $y'' - 2y = 0$.

a) Find the general solution of the differential equation. **[3 marks]**

b) Hence, show that the solution which satisfies the boundary conditions

$$y\,(0) = 1 \text{ and } y'\left(\frac{\sqrt{2}}{2}\right) = 0 \text{ is}$$

$$y = \frac{1}{e^2 + 1}\left(e^{\sqrt{2}\,x} + e^{2-\sqrt{2}\,x}\right).$$

[7 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

FORM TP 2016282

TEST CODE **02234020**

MAY/JUNE 2016

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PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

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SECTION A

Module 1

Answer BOTH questions.

1. (a) A quadratic equation is given by $ax^2 + bx + c = 0$, where $a, b, c \in \mathbf{R}$. The complex roots of the equation are $\alpha = 1 - 3\mathrm{i}$ and β.

(i) Calculate $(\alpha + \beta)$ and $(\alpha\beta)$.

[3 marks]

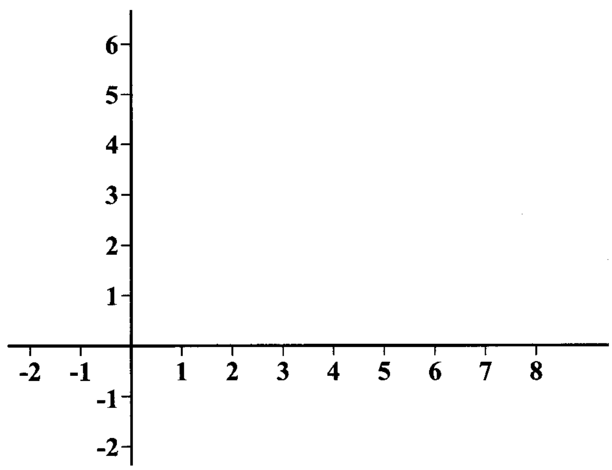
GO ON TO THE NEXT PAGE

(ii) Hence, show that an equation with roots $\frac{1}{\alpha - 2}$ and $\frac{1}{\beta - 2}$ is given by $10x^2 + 2x + 1 = 0$.

[6 marks]

(b) Two complex numbers are given as $u = 4 + 2\text{i}$ and $v = 1 + 2\sqrt{2}\text{i}$.

(i) Complete the Argand diagram below to illustrate u.



[1 mark]

(ii) On the same Argand plane, sketch the circle with equation $|z - u| = 3$.

[2 marks]

(iii) Calculate the modulus and principal argument of $z = \left(\frac{u}{v}\right)^5$.

[6 marks]

(c) A function f is defined by the parametric equations

$$x = 4 \cos t \text{ and } y = 3 \sin 2t \text{ for } 0 \leq t \leq \pi.$$

Determine the x-coordinates of the two stationary values of f.

[7 marks]

Total 25 marks

2. (a) A function w is defined as $w(x, y) = \ln\left|\dfrac{2x + y}{x - 10}\right|$.

Determine $\dfrac{\partial w}{\partial x}$.

[4 marks]

(b) Determine $\int e^{2x} \sin e^{x} \, dx$.

[6 marks]

(c) Let $f(x) = \dfrac{x^2 + 2x + 3}{(x - 1)(x^2 + 1)}$ for $2 \leq x \leq 5$.

(i) Use the trapezium rule with three equal intervals to estimate the area bounded by f and the lines $y = 0$, $x = 2$ and $x = 5$.

[5 marks]

GO ON TO THE NEXT PAGE

(ii) Using partial fractions, show that $f(x) = \frac{3}{x-1} - \frac{2x}{x^2+1}$.

[6 marks]

GO ON TO THE NEXT PAGE

(iii) Hence, determine the value of $\int_{2}^{5} f(x)\, dx$.

[4 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) A sequence is defined by the recurrence relation $u_{n+1} = u_{n-1} + x(u_n)'$, where $u_1 = 1$, $u_2 = x$ and $(u_n)'$ is the derivative of u_n.

For example, $u_3 = u_{2+1} = u_1 + x(u_2)' = 1 + x$.

Given that $u_8 = 13x + 1$ and that $u_{10} = 34x + 1$, find $(u_9)'$.

[4 marks]

GO ON TO THE NEXT PAGE

(b) The n^{th} partial sum of a series, S_n, is given by $S_n = \sum_{r=1}^{n} r(r-1)$.

(i) Show that $S_n = \dfrac{n(n^2-1)}{3}$.

[7 marks]

(ii) Hence, or otherwise, evaluate $\sum_{10}^{20} r(r-1)$.

[5 marks]

GO ON TO THE NEXT PAGE

(c) (i) Given that ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$ show that $\dfrac{{}^{2r}P_{r}\,{}^{n}P_{r}}{(2r)!}$ is equal to the binomial coefficient ${}^{n}C_{r}$.

[4 marks]

GO ON TO THE NEXT PAGE

(ii) Determine the coefficient of the term in x^3 in the binomial expansion of $(3x + 2)^5$.

[5 marks]

Total 25 marks

4. (a) The function f is defined as $f(x) = \sqrt[6]{4x^2 + 4x + 1}$ for $-1 < x < 1$.

(i) Show that $f(x) = (1 + 2x)^{\frac{1}{3}}$.

[3 marks]

The series expansion of $(1+x)^k$ is given as

$$1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \frac{k(k-1)(k-2)(k-3)x^4}{4!} + \ldots$$

where $k \in \mathbf{R}$ and $-1 < x < 1$.

(ii) Determine the series expansion of f up to and including the term in x^4.

[5 marks]

(iii) Hence, approximate f(0.4) correct to 2 decimal places.

[3 marks]

GO ON TO THE NEXT PAGE

(b) The function $h(x) = x^3 + x - 1$ is defined on the interval [0, 1].

(i) Show that $h(x) = 0$ has a root on the interval [0, 1].

[3 marks]

GO ON TO THE NEXT PAGE

(ii) Use the iteration $x_{n+1} = \dfrac{1}{x_n^{\,2} + 1}$ with initial estimate $x_1 = 0.7$ to estimate the root of h correct to 2 decimal places.

[6 marks]

(c) Use two iterations of the Newton–Raphson method with initial estimate $x_1 = 1$ to approximate the root of the equation $g(x) = e^{4x-3} - 4$ in the interval [1, 2]. Give your answer correct to 3 decimal places.

[5 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

SECTION C

Module 3

Answer BOTH questions.

5. (a) A bus has 13 seats for passengers. Eight passengers boarded the bus before it left the terminal.

(i) Determine the number of possible seating arrangements of the passengers who boarded the bus at the terminal.

[2 marks]

(ii) At the first stop, no passengers will get off the bus but there are eight other persons waiting to board the same bus. Among those waiting are three friends who must sit together.

Determine the number of possible groups of five of the waiting passengers that can join the bus.

[4 marks]

GO ON TO THE NEXT PAGE

(b) Gavin and his best friend Alexander are two of the five specialist batsmen on his school's cricket team.

Given that the specialist batsmen must bat before the non-specialist batsmen and that all five specialist batsmen may bat in any order, what is the probability that Gavin and Alexander are the opening pair for a given match?

[5 marks]

(c) A matrix **A** is given as

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 4 & 3 \\ -1 & 6 & 0 \end{bmatrix}$$

(i) Find the |**A**|, determinant of **A**.

[4 marks]

(ii) Hence, or otherwise, find $\mathbf{A}^{-1}$, the inverse of $\mathbf{A}$.

[10 marks]

Total 25 marks

6. (a) Two fair coins and one fair die are tossed at the same time.

(i) Calculate the number of outcomes in the sample space.

[3 marks]

(ii) Find the probability of obtaining exactly one head.

[2 marks]

(iii) Calculate the probability of obtaining at least one head on the coins and an even number on the die on a particular attempt.

[4 marks]

(b) Determine whether $y = C_1x + C_2x^2$ is a solution to the differential equation

$$\frac{x^2}{2}y'' - xy' + y = 0, \text{ where } C_1 \text{ and } C_2 \text{ are constants.}$$

[6 marks]

(c) (i) Show that the general solution to the differential equation

$$3\,(x^2 + x)\,\frac{\mathrm{d}y}{\mathrm{d}x} = 2y\,(1 + 2x) \text{ is}$$

$$y = C\sqrt[3]{(x^2 + x)^2}, \text{ where } C \in \mathbf{R}$$

[7 marks]

GO ON TO THE NEXT PAGE

(ii) Hence, given that $y(1) = 1$, solve $3(x^2 + x)\dfrac{dy}{dx} = 2y(1 + 2x)$.

[3 marks]

Total 25 marks

END OF TEST

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EXTRA SPACE

If you use this extra page, you MUST write the question number clearly in the box provided.

Question No. $\boxed{}$

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MAY/JUNE 2017

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SECTION A

Module 1

Answer BOTH questions.

1. (a) Find the first derivative of the function $f(x) = \cos^{-1}(\sin^{-1} x)$.

[3 marks]

GO ON TO THE NEXT PAGE

(b) A function, w, is defined as $w(x, y) = \ln\left|\frac{2x + y}{x - 1}\right|$.

(i) Given that $\frac{\partial w}{\partial x} = -\frac{1}{9}$ at the point $(4, y_0)$, calculate the value of y_0.

[5 marks]

GO ON TO THE NEXT PAGE

(ii) Show that $\dfrac{\partial^2 w}{\partial y \partial x} - 2\,\dfrac{\partial^2 w}{\partial y^2} = 0.$

[5 marks]

GO ON TO THE NEXT PAGE

(c) (i) Find the complex numbers $u = x + iy$ such that x and y are real and $u^2 = -15 + 8i$.

[7 marks]

GO ON TO THE NEXT PAGE

(ii) Hence, or otherwise, solve the equation $z^2 - (3 + 2i)\,z + (5 + i) = 0$, for z .

[5 marks]

Total 25 marks

GO ON TO THE NEXT PAGE

2. (a) (i) Use integration by parts to derive the reduction formula

$$aI_n = x^n e^{ax} - nI_{n-1} \text{ where } I_n = \int x^n e^{ax}\, dx .$$

[4 marks]

(ii) Hence, or otherwise, determine $\int x^3 e^{3x}\, dx$.

[6 marks]

(b) Calculate $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx$.

[5 marks]

(c) (i) Use partial fractions to show that

$$\frac{2x^2 - x + 4}{x^3 + 4x} = \frac{1}{x} + \frac{x}{x^2 + 4} - \frac{1}{x^2 + 4}.$$

[5 marks]

(ii) Hence, or otherwise, determine $\int \frac{2x^2 - x + 4}{x^3 + 4x}\, dx$.

[5 marks]

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Determine the Taylor series expansion about $x = 2$ of the function $f(x) = \ln(5 + x)$ up to and including the term in x^3.

[6 marks]

(ii) Hence, obtain an approximation for $f(7) - \ln(7)$.

[2 marks]

(b) (i) Use mathematical induction to prove that

$$1^3 + 2^3 + \ldots + n^3 = \frac{1}{4} n^2 (n + 1)^2, \text{ for } n \in \mathbf{N}.$$

[9 marks]

(ii) Hence, or otherwise, show that $\sum_{i=1}^{2n+1} i^3 = (2n+1)^2(n+1)^2$.

[3 marks]

(iii) Use the results of Parts (b) (i) and (ii) to show that

$$\sum_{i=1}^{n+1} (2i-1)^3 = (n+1)^2(2n^2+4n+1).$$

[5 marks]
Total 25 marks

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4. (a) Eight boys and two girls are to be seated on a bench. How many seating arrangements are possible if the girls can neither sit together nor sit at the ends?

[5 marks]

(b) (i) Show that the binomial expansion of $(1 + \frac{1}{8}x)^8$ up to and including the term in x^4 is

$$1 + x + \frac{7}{16}x^2 + \frac{7}{64}x^3 + \frac{35}{2048}x^4 .$$

[4 marks]

(ii) Use the expansion to approximate the value of $(1.0125)^8$.

[4 marks]

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(c) (i) Use the intermediate value theorem to show that $f(x) = \sqrt{x} - \cos x$ has a root in the interval [0, 1].

[3 marks]

(ii) Use two iterations of the interval bisection method to approximate the root of f in the interval [0, 1].

[4 marks]

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(d) (i) Show that $x_{n+1} = \sqrt[3]{\frac{9-3x_n}{2}}$ is an appropriate iterative formula for finding the root of $f(x) = -2x^3 - 3x + 9$.

[2 marks]

(ii) Apply the iterative formula with initial approximation $x_1 = 1$, to obtain a third approximation, x_3, of the root of the equation.

[3 marks]

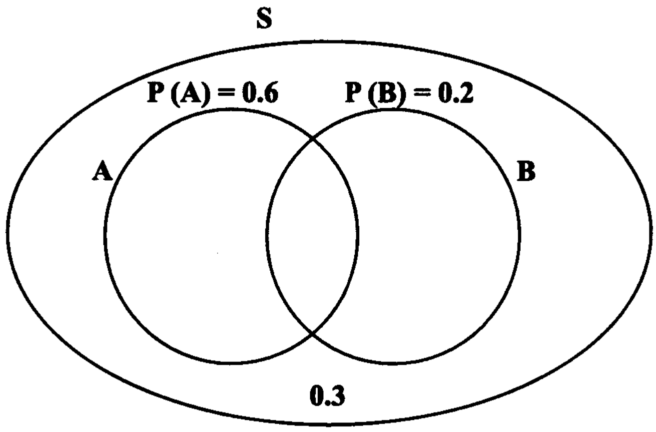
Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) The following Venn diagram shows a sample space, S, and the probabilities of two events, A and B, within the sample space S.



(i) Given that $P(A \cup B) = 0.7$, calculate P (A only).

[3 marks]

(ii) Hence, determine whether events A and B are independent. **Justify your answer.**

[2 marks]

(b) Two balls are to be drawn at random without replacement from a bag containing 3 red balls, 2 blue balls and 1 white ball.

(i) Represent the outcomes of the draws and their corresponding probabilities on a tree diagram.

[4 marks]

(ii) Determine the probability that the second ball drawn is white.

[3 marks]

(c) Given the matrices $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 1 & 3 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 30 & -12 & 2 \\ 5 & -8 & 3 \\ -5 & 4 & 1 \end{pmatrix}$,

(i) show that $AB = 20I$.

[5 marks]

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(ii) Hence, deduce the inverse, A^{-1}, of the matrix A.

[2 marks]

(iii) Hence, or otherwise, solve the system of linear equations given by

$$
\begin{aligned}
x - y + z &= 1 \\
x - 2y + 4z &= 5 \\
x + 3y + 9z &= 25
\end{aligned}
$$

[6 marks]

Total 25 marks

6. (a) (i) Find the general solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} + 2xy = \sqrt[3]{x}$.

[7 marks]

(ii) Hence, given that $y = 2$ when $x = 0$, calculate $y\,(1)$.

[3 marks]

(b) (i) Use the substitution $u = y'$ to show that the differential equation $y'' + 4y' = 2\cos 3x - 4\sin 3x$ can be reduced to $u' + 4u = 2\cos 3x - 4\sin 3x$.

[2 marks]

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(ii) Hence, or otherwise, find the general solution of the differential equation.

[13 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.