Parabolas with Vertices Not at the Origin

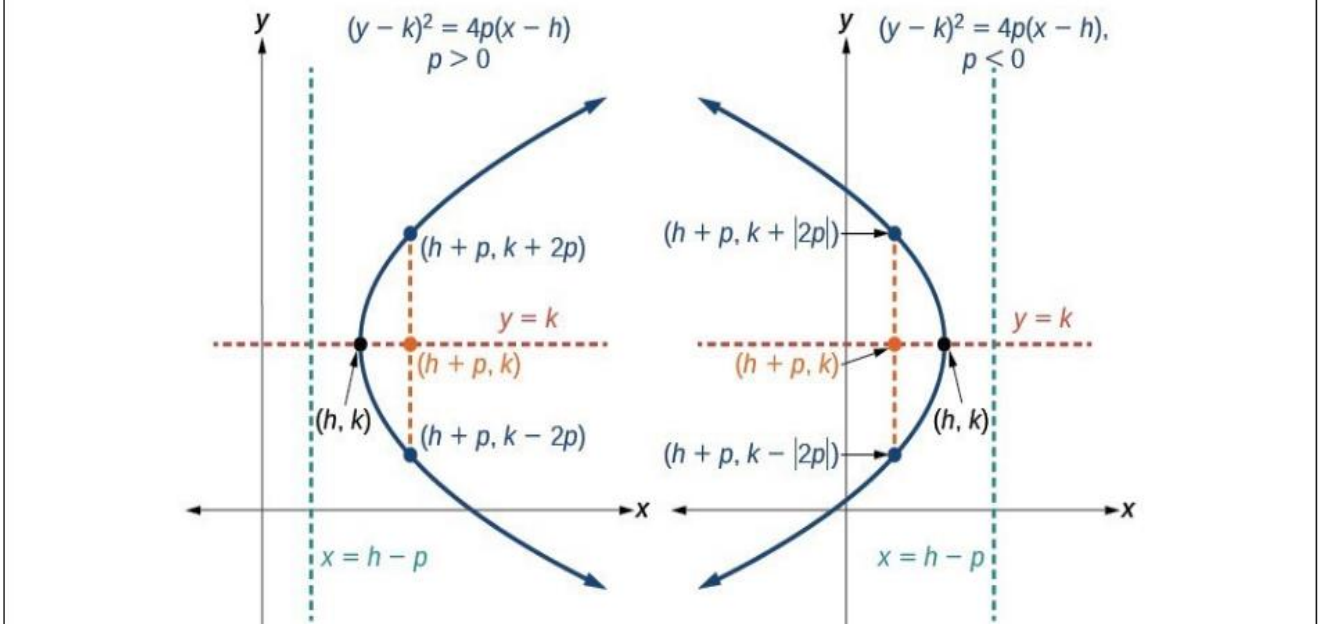
LEARNING OUTCOMES

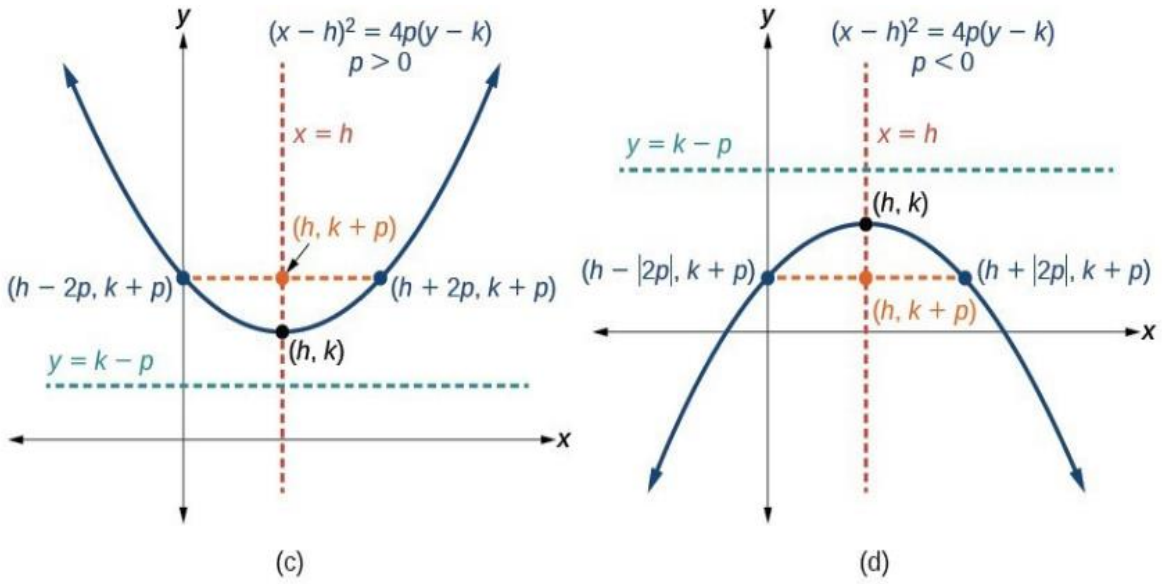
- Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the focal diameter given the equation of a parabola in standard form.
- Find the equation of a parabolic shaped object given dimensions.

Like other graphs we've worked with, the graph of a parabola can be translated. If a parabola is translated h units horizontally and k units vertically, the vertex will be (h, k). This translation results in the standard form of the equation we saw previously with x replaced by $(x - h)$ and y replaced by $(y - k)$.

To graph parabolas with a vertex (h, k) other than the origin, we use the standard form $(y - k)^2 = 4p(x - h)$ for parabolas that have an axis of symmetry parallel to the *x*-axis, and $(x - h)^2 = 4p(y - k)$ for parabolas that have an axis of symmetry parallel to the *y*-axis. These standard forms are given below, along with their general graphs and key features.

Axis of Symmetry	Equation	Focus	Directrix	Endpoints of focal diameter
$y = k$	$(y - k)^2 = 4p(x - h)$	$(h + p,\ k)$	$x = h - p$	$(h + p,\ k \pm 2p)$
$x = h$	$(x - h)^2 = 4p(y - k)$	$(h,\ k + p)$	$y = k - p$	$(h \pm 2p,\ k + p)$





(a) When $p > 0$, the parabola opens right. (b) When $p < 0$, the parabola opens left. (c) When $p > 0$, the parabola opens up. (d) When $p < 0$, the parabola opens down.

EXAMPLE: GRAPHING A PARABOLA WITH VERTEX (*H*, *K*) AND AXIS OF SYMMETRY PARALLEL TO THE *X*-AXIS

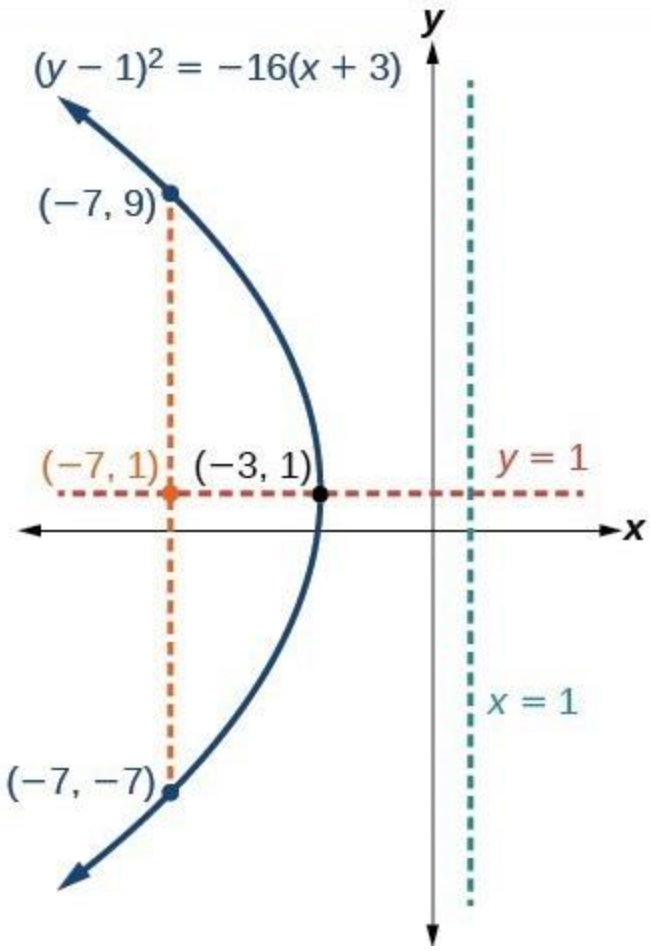
Graph $(y-1)^2 = -16(x+3)$. Identify and label the **vertex**, **axis of symmetry**, **focus**, **directrix**, and endpoints of the **focal diameter**.

Show Solution

The standard form that applies to the given equation is $(y-k)^2 = 4p(x-h)$. Thus, the axis of symmetry is parallel to the *x*-axis. It follows that:

- the vertex is $(h, k) = (-3, 1)$
- the axis of symmetry is $y = k = 1$
- $-16 = 4p$, so $p = -4$. Since $p < 0$, the parabola opens left.
- the coordinates of the focus are $(h + p, k) = (-3 + (-4), 1) = (-7, 1)$
- the equation of the directrix is $x = h - p = -3 - (-4) = 1$
- the endpoints of the focal diameter are $(h + p, k \pm 2p) = (-3 + (-4), 1 \pm 2(-4))$, or $(-7, -7)$ and $(-7, 9)$

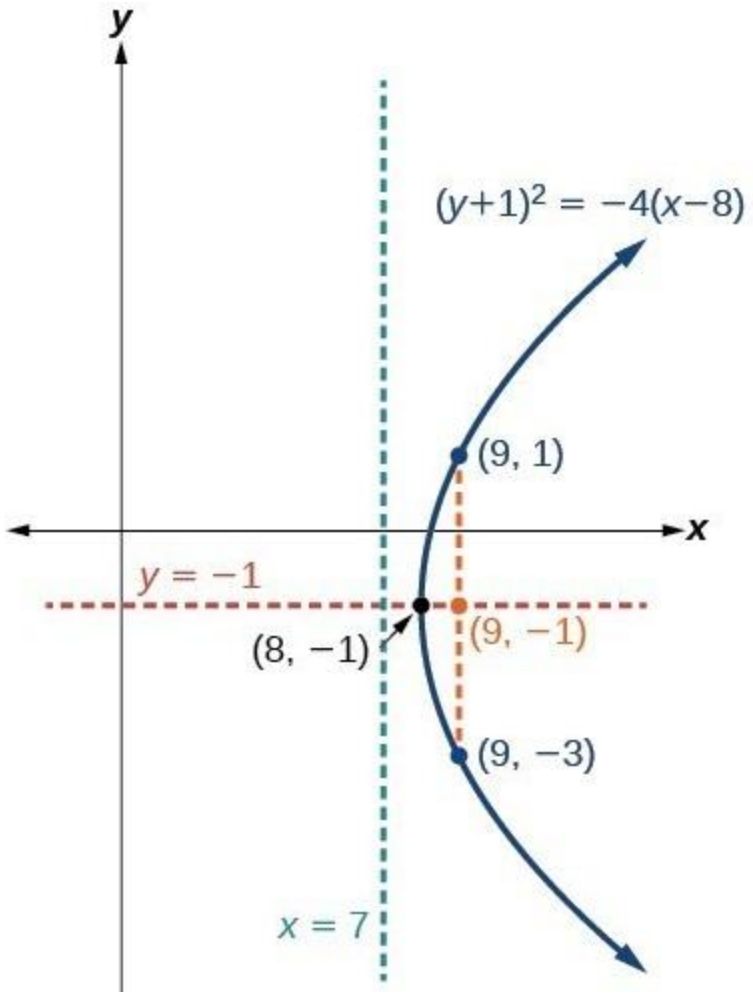
Next we plot the vertex, axis of symmetry, focus, directrix, and focal diameter, and draw a smooth curve to form the parabola.



Graph $(y+1)^2 = 4(x-8)$. Identify and label the vertex, axis of symmetry, focus, directrix, and endpoints of the focal diameter.

Show Solution

Vertex: $(8, -1)$; Axis of symmetry: $y = -1$; Focus: $(9, -1)$; Directrix: $x = 7$; Endpoints of the latus rectum: $(9, -3)$ and $(9, 1)$.


y
x
$(y+1)^2 = -4(x-8)$
(9, 1)
$y = -1$
(8, −1)
(9, −1)
(9, −3)
$x = 7$