Section 5.4 More Trigonometric Graphs

Graphs of the Tangent, Cotangent, Secant, and Cosecant Function

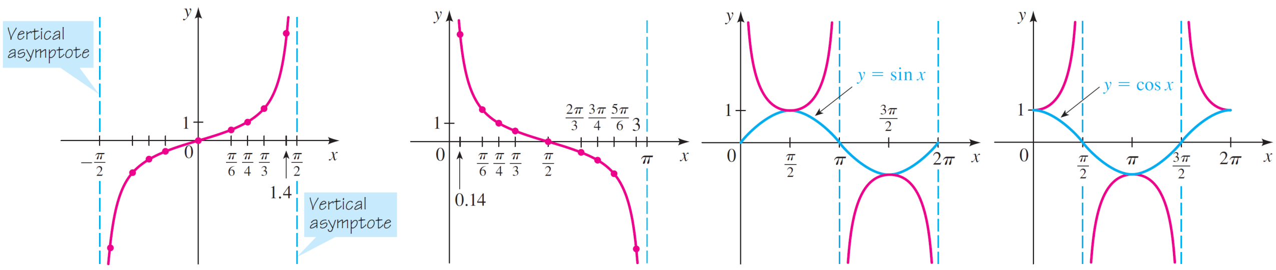
Periodic Properties

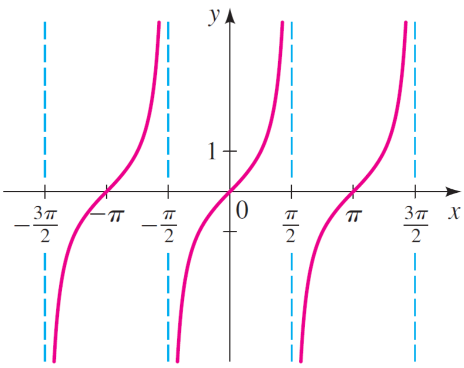
The functions tangent and cotangent have period π:

$$\tan(x + \pi) = \tan x \qquad \cot(x + \pi) = \cot x$$

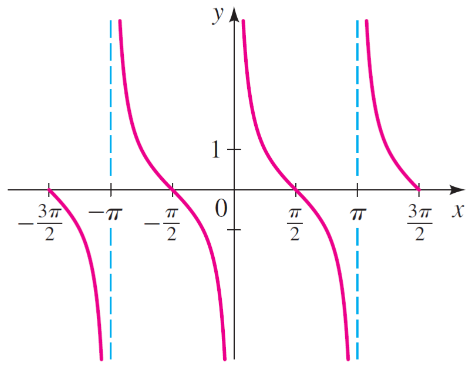
The functions cosecant and secant have period 2π:

$$\csc(x + 2\pi) = \csc x \qquad \sec(x + 2\pi) = \sec x$$

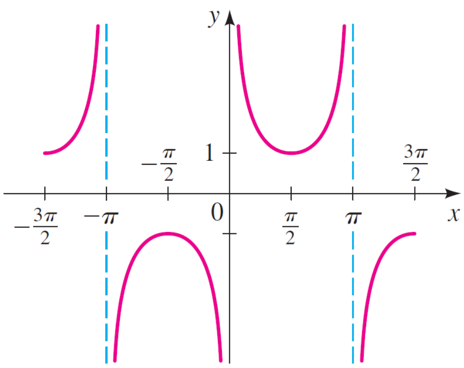




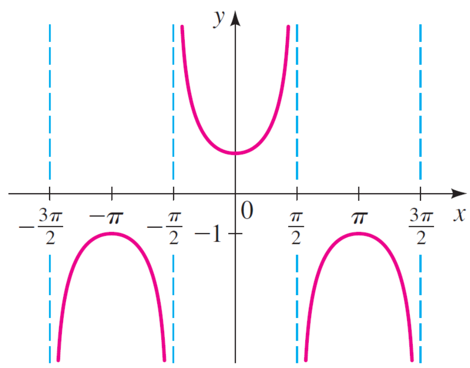
(a) $y = \tan x$



(b) $y = \cot x$

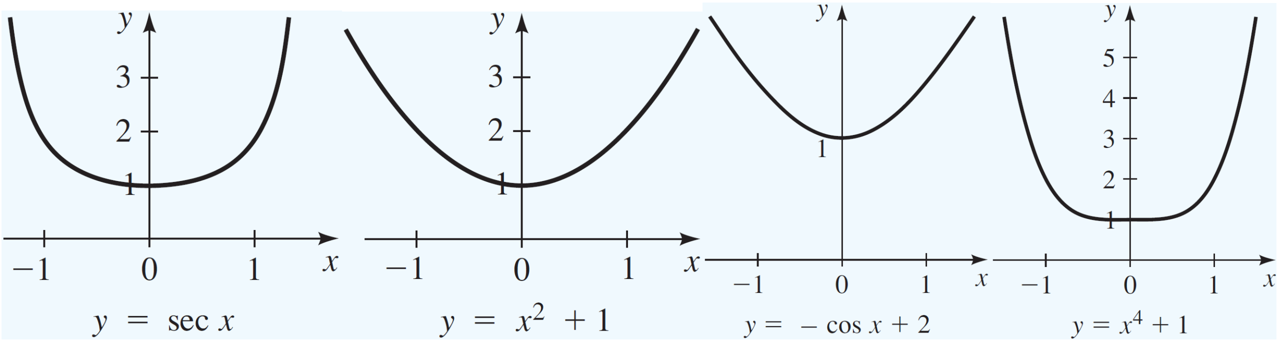


(c) $y = \csc x$

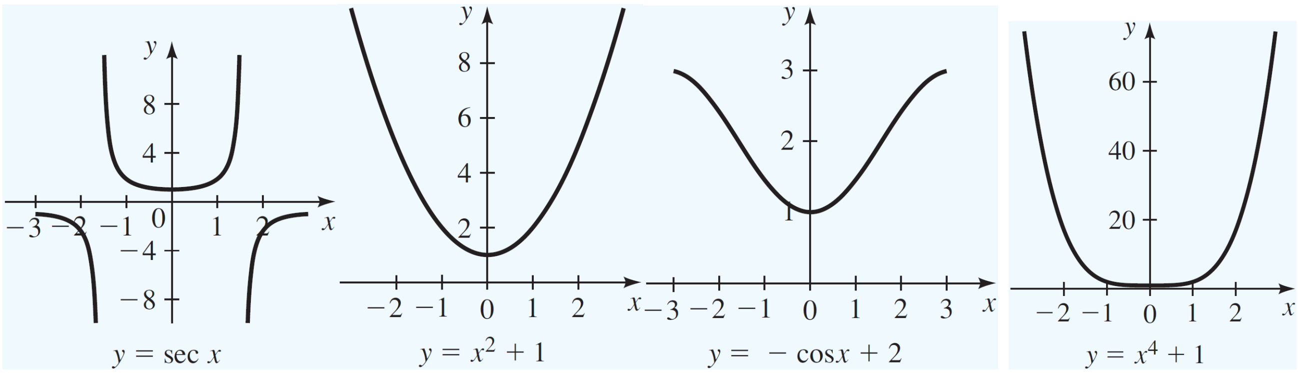


(d) $y = \sec x$

REMARK: Many curves have a "U" shape near zero. For example, notice that the functions $\sec x$ and x^2+1 are very different functions. But when we casually sketch their graphs near $x=0$ the sketches look similar:



However, if we look at their behavior on a larger interval, then their similarity decreases, but some of them would still be hard to tell apart just by looking at the shapes of the curves.



Graphs Involving Tangent and Cotangent Functions

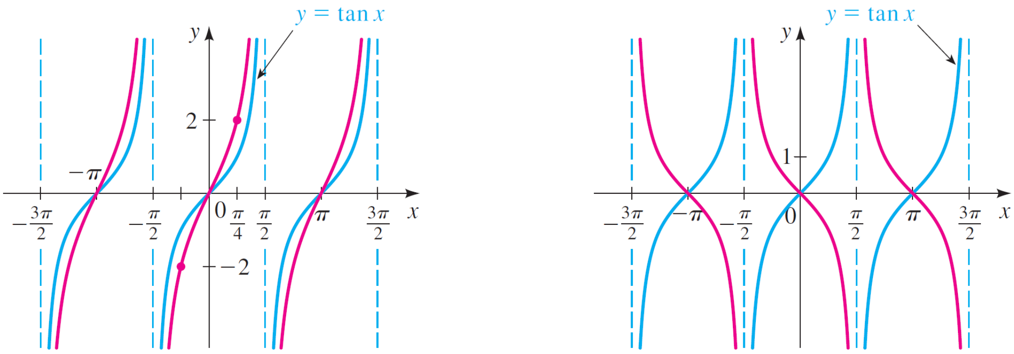
EXAMPLE: Graph each function.

(a) $y=2\tan x$ (b) $y=-\tan x$

Solution:

(a) To graph $y=2\tan x$, we multiply the y-coordinate of each point on the graph of $y=\tan x$ by 2.

(b) The graph of $y=-\tan x$ is obtained from that of $y=\tan x$ by reflecting in the x-axis.



(a) $y=2\tan x$ (b) $y=-\tan x$

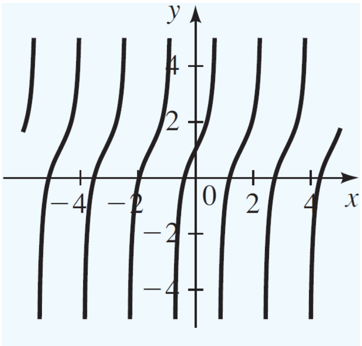
Tangent and Cotangent Curves

The functions

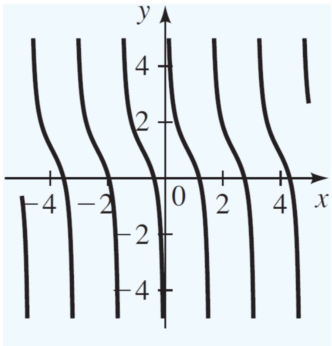
$$y = a \tan kx \qquad \text{and} \qquad y = a \cot kx \qquad (k > 0)$$

have period π/k.

EXAMPLE: Graphs of variants of trigonometric curves $y = \tan 2x + 1$ and $y = \cot 2x + 1$ have period $\dfrac{\pi}{2}$ and vertical shift 1.



$y = \tan 2x + 1$



$y = \cot 2x + 1$

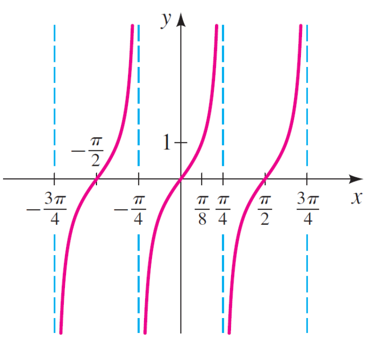
EXAMPLE: Graph each function.

(a) $y = \tan 2x$ (b) $y = \tan 2\left(x - \dfrac{\pi}{4}\right)$

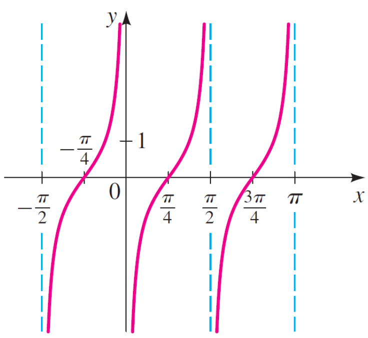
Solution:

(a) The period is $\pi \div 2 = \pi/2$ and an appropriate interval is $(-\pi/4, \pi/4)$. The endpoints $x = -\pi/4$ and $x = \pi/4$ are vertical asymptotes. Thus, we graph one complete period of the function on $(-\pi/4, \pi/4)$. The graph has the same shape as that of the tangent function, but is shrunk horizontally by a factor of $1/2$. We then repeat that portion of the graph to the left and to the right.

(b) The graph is the same as that in part (a), but it is shifted to the right $\pi/4$.



(a) $y = \tan 2x$



(b) $y = \tan 2\left(x - \frac{\pi}{4}\right)$

EXAMPLE: Graph each function.

(a) $y = \tan \dfrac{x}{2}$ (b) $y = \tan \pi x$ (c) $y = -3 \tan 2x$

EXAMPLE: Graph each function.

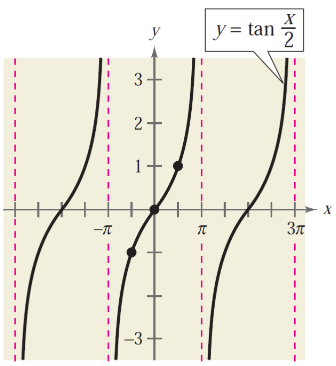
(a) $y = \tan \dfrac{x}{2}$ (b) $y = \tan \pi x$ (c) $y = -3 \tan 2x$

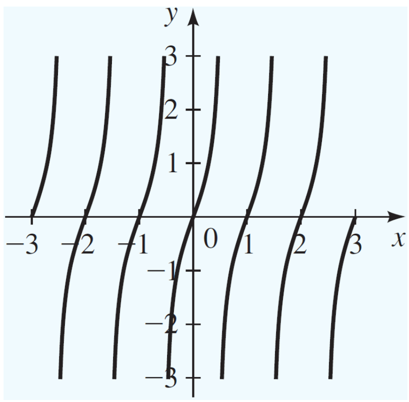
Solution:

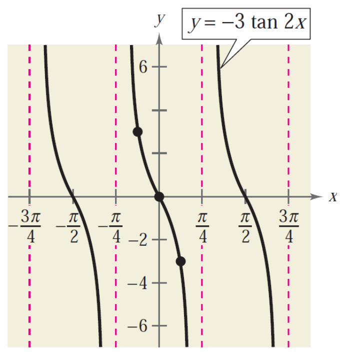
(a) The period is $\pi \div \frac{1}{2} = 2\pi$ and an appropriate interval is $(-\pi, \pi)$. The endpoints $x = -\pi$ and $x = \pi$ are vertical asymptotes. Thus, we graph one complete period of the function on $(-\pi, \pi)$. The graph has the same shape as that of the tangent function, but is stretched horizontally by a factor of 2. We then repeat that portion of the graph to the left and to the right.

(b) The period is $\pi \div \pi = 1$ and an appropriate interval is $(-1/2, 1/2)$. The endpoints $x = -1/2$ and $x = 1/2$ are vertical asymptotes. Thus, we graph one complete period of the function on $(-1/2, 1/2)$. The graph has the same shape as that of the tangent function, but is shrunk horizontally by a factor of $1/\pi$. We then repeat that portion of the graph to the left and to the right.

(c) The period is $\pi \div 2 = \pi/2$ and an appropriate interval is $(-\pi/4, \pi/4)$. The endpoints $x = -\pi/4$ and $x = \pi/4$ are vertical asymptotes. Thus, we graph one complete period of the function on $(-\pi/4, \pi/4)$. The graph has the same shape as that of the tangent function, but is shrunk horizontally by a factor of $1/2$, stretched vertically by a factor of 3 and reflected about the x-axis. We then repeat that portion of the graph to the left and to the right.







EXAMPLE: Graph each function.

(a) $y = 2 \cot \dfrac{x}{3}$ (b) $y = 2 \cot \left(3x - \dfrac{\pi}{2}\right)$ (c) $y = 3 \cot (\pi x - \pi)$

EXAMPLE: Graph each function.

(a) $y = 2\cot\dfrac{x}{3}$ (b) $y = 2\cot\left(3x - \dfrac{\pi}{2}\right)$ (c) $y = 3\cot(\pi x - \pi)$

Solution:

(a) The period is $\pi \div \frac{1}{3} = 3\pi$ and an appropriate interval is $(0, 3\pi)$. The endpoints $x = 0$ and $x = 3\pi$ are vertical asymptotes. Thus, we graph one complete period of the function on $(0, 3\pi)$. The graph has the same shape as that of the cotangent function, but is stretched horizontally by a factor of 3 and stretched vertically by a factor of 2. We then repeat that portion of the graph to the left and to the right.

(b) We first put this in the form $y = a\cot k(x - b)$ by factoring 3 from the expression $3x - \dfrac{\pi}{2}$:

$$y = 2\cot\left(3x - \frac{\pi}{2}\right) = 2\cot 3\left(x - \frac{\pi}{6}\right)$$

Thus the graph is the same as that of $y = 2\cot 3x$ but is shifted to the right $\pi/6$. The period of $y = 2\cot 3x$ is $\pi \div 3 = \pi/3$, and an appropriate interval is $(0, \pi/3)$. To get the corresponding interval for the desired graph, we shift this interval to the right $\pi/6$. This gives

$$\left(0 + \frac{\pi}{6}, \frac{\pi}{3} + \frac{\pi}{6}\right) = \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$$

One can get the same result in an other way: Since the period of $y = \cot x$ is π, the function $y = 2\cot\left(3x - \dfrac{\pi}{2}\right)$ will go through one complete period as $3x - \dfrac{\pi}{2}$ varies from 0 to π.

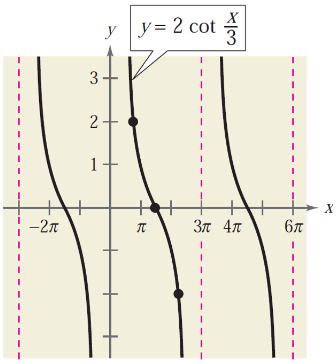
Start of period:	End of period:
$3x - \dfrac{\pi}{2} = 0$	$3x - \dfrac{\pi}{2} = \pi$
$3x = \dfrac{\pi}{2}$	$3x = \dfrac{3\pi}{2}$
$x = \dfrac{\pi}{6}$	$x = \dfrac{\pi}{2}$

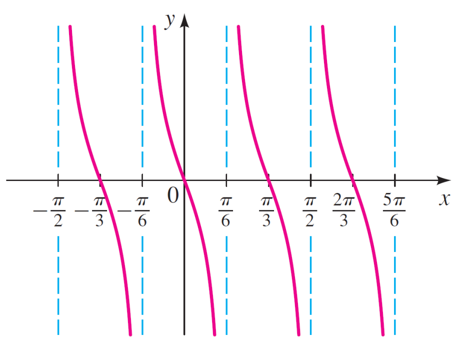
Finally, we graph one period in the shape of cotangent on the interval $(\pi/6, \pi/2)$ and repeat that portion of the graph to the left and to the right.

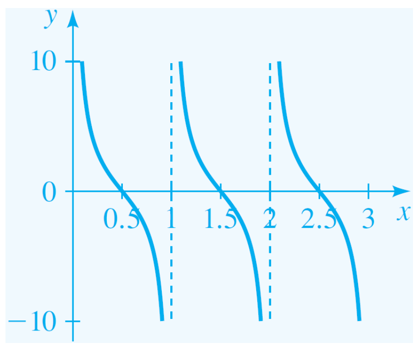
(c) We first put this in the form $y = a\cot k(x - b)$ by factoring π from the expression $\pi x - \pi$:

$$y = 3\cot(\pi x - \pi) = 3\cot\pi(x - 1)$$

Thus the graph is the same as that of $y = 3\cot\pi x$ but is shifted to the right 1. The period of $y = 3\cot\pi x$ is $\pi \div \pi = 1$, and an appropriate interval is $(0, 1)$. Since the length of this interval and the shift are both 1, we graph one period in the shape of cotangent on the interval $(0, 1)$ and repeat that portion of the graph to the left and to the right.







Graphs Involving the Cosecant and Secant Functions

Cosecant and Secant Curves

The functions

$$y = a \csc kx \qquad \text{and} \qquad y = a \sec kx \qquad (k > 0)$$

have period $2\pi/k$.

EXAMPLE: Graph each function.

(a) $y = \dfrac{1}{2}\csc 2x$ (b) $y = \dfrac{1}{2}\csc\left(2x + \dfrac{\pi}{2}\right)$

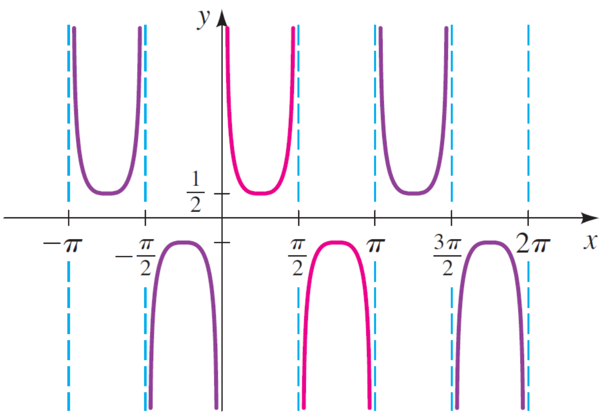
Solution:

(a) The period is $2\pi \div 2 = \pi$. An appropriate interval is $[0, \pi]$, and the asymptotes occur in this interval whenever $\sin 2x = 0$. So the asymptotes in this interval are $x = 0$, $x = \pi/2$, and $x = \pi$. With this information we sketch on the interval $[0, \pi]$ a graph with the same general shape as that of one period of the cosecant function.

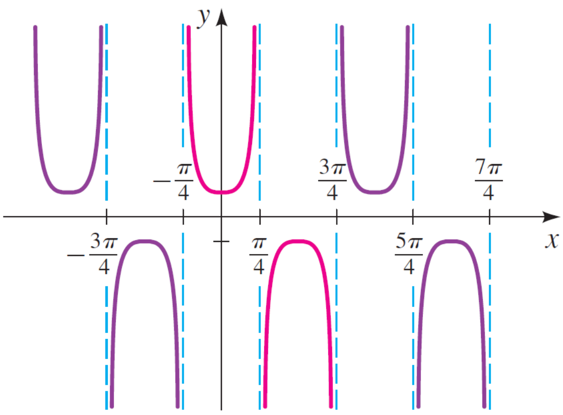
(b) We first write

$$y = \frac{1}{2}\csc\left(2x + \frac{\pi}{2}\right) = \frac{1}{2}\csc 2\left(x + \frac{\pi}{4}\right)$$

From this we see that the graph is the same as that in part (a), but shifted to the left $\pi/4$.



(a) $y = \frac{1}{2}\csc 2x$



(b) $y = \frac{1}{2}\csc\left(2x + \frac{\pi}{2}\right)$

EXAMPLE: Graph each function.

(a) $y = \csc 2x$ (b) $y = 2\csc\left(x + \dfrac{\pi}{4}\right)$ (c) $y = 3\sec \dfrac{1}{2}x$

EXAMPLE: Graph each function.

(a) $y = \sec 2x$ (b) $y = 2\csc\left(x + \dfrac{\pi}{4}\right)$ (c) $y = 3\sec\dfrac{1}{2}x$

Solution:

(a) The period is $2\pi \div 2 = \pi$. An appropriate interval is $[-\pi/2, \pi/2]$, and the asymptotes occur in this interval wherever $\cos 2x = 0$. Thus, the asymptotes in this interval are $x = -\pi/4,\ x = \pi/4$. With this information we sketch on the interval $[-\pi/2, \pi/2]$ a graph with the same general shape as that of one period of the secant function. The complete graph in the Figure below is obtained by repeating this portion of the graph to the left and to the right.

(b) The period is $2\pi \div 1 = 2\pi$. An appropriate interval is

$$\left[0 - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}\right] = \left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$$

and the asymptotes occur in this interval wherever $\sin\left(x + \dfrac{\pi}{4}\right) = 0$. Thus, the asymptotes in this interval are $x = \pi/4,\ x = 3\pi/4$, and $x = 7\pi/4$. With this information we sketch on the interval $[-\pi/4, 7\pi/4]$ a graph with the same general shape as that of one period of the secant function. The complete graph in the Figure below is obtained by repeating this portion of the graph to the left and to the right.

(c) The period is $2\pi \div \frac{1}{2} = 4\pi$. An appropriate interval is $[0, 4\pi]$, and the asymptotes occur in this interval wherever $\cos\frac{1}{2}x = 0$. Thus, the asymptotes in this interval are $x = \pi,\ x = 3\pi$. With this information we sketch on the interval $[0, 4\pi]$ a graph with the same general shape as that of one period of the secant function. The complete graph in the Figure below is obtained by repeating this portion of the graph to the left and to the right.