FORM TP 02134020/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGERBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 02

2 hours 30 minutes

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each Module is 50.
The maximum mark for this examination is 150.
This examination consists of 6 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Ruler and graph paper

SECTION A (MODULE 1)

Answer BOTH questions.

1. (a) Let **p** and **q** be given propositions.

(i) Copy and complete the table below to show the **truth tables** of $p \rightarrow q$ and $\sim p \vee q$.

[3 marks]

p	q	~p	$p \rightarrow q$	$\sim p \vee q$

(ii) Hence, state whether the compound propositions $p \rightarrow q$ and $\sim p \vee q$ are logically equivalent, stating reasons for your answer.

[2 marks]

(iii) Use the **algebra of propositions** to show that $p \wedge (p \rightarrow q) = p \wedge q$.

[3 marks]

(b) The binary operation * is defined on the set of real numbers, $\mathbb{R}$, as follows:
$x * y = x + y - 1$
For all x, y in $\mathbb{R}$

Prove that

(i) * is closed in $\mathbb{R}$, **[3 marks]**

(ii) * is commutative in $\mathbb{R}$, **[2 marks]**

(iii) *is associative in $\mathbb{R}$. **[4 marks]**

(c) Let $y = \dfrac{2x}{x^2 + 4}$.

(i) Show that for all real values of x, $-\frac{1}{2} \leq y \leq \frac{1}{2}$ **[5 marks]**

(ii) Hence, sketch the graph of y for all x such that $-2 \leq x \leq 2$

[3 marks]

Total 25 marks

2. (a) Two of the roots of the cubic equation $2x^3 + px^2 + qx + 2 = 0$ are -1 and $\frac{1}{2}$.

Find

(i) the values of the constants p and q **[4 marks]**

(ii) the third root of the equation. **[3 marks]**

(b) Prove by Mathematical Induction that $\sum_{r=1}^{n} (6r + 5) = n(3n + 8)$. **[10 marks]**

(c) Solve for x the following equation $e^{2x} + 2e^{-2x} = 3$.

[8 marks]

Total 25 marks

SECTION B (MODULE 2)

Answer BOTH questions.

3. (a) (i) Prove the identity $\dfrac{\sin 3\theta + \sin\theta}{\cos 3\theta + \cos\theta} \equiv \tan 2\theta$. **[4 marks]**

(ii) Solve the equation $\sin\theta + \sin 2\theta + \sin 3\theta = 0$, $0 \leq \theta \leq 2\pi$. **[7 marks]**

(b) (i) Express $f(\theta) = 8\cos\theta + 6\sin\theta$ in the form $r\cos(\theta - \alpha)$ where $r > 0$, $0^0 < \alpha < 90^0$. **[3 marks]**

(ii) Determine the minimum value of

$$g(\theta) = \frac{10}{10 + 8\cos\theta + 6\sin\theta}$$ and

state the value of θ for which $g(\theta)$ is a minimum. **[4 marks]**

(c) Let A = (2, 0, 0), B = (0, 0, 2), C = (0, 2, 0).

(i) Express the vectors $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in the form $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ **[2marks]**

(ii) Show that the vector $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ is perpendicular to the plane through A, B, and C. **[2 marks]**

(iii) Hence, find the Cartesian equation of the plane through A, B and C. **[3 marks]**

Total 25 marks

4. The equation of the line L is $x + 2y = 7$ and the equation of the circle C is $x^2 + y^2 - 4x - 1 = 0$.

(a) Show that the line L is a tangent to the circle C. **[9 marks]**

(b) Find,

(i) the equation of the tangent M diametrically opposite to the tangent L of circle C. **[5 marks]**

(ii) the equation of the diameter of C parallel to L and the coordinates of its points of intersection with C. **[6 marks]**

(c) The parametric equations of a curve, C, are given by

$$x = \frac{t}{1+t} \quad \text{and} \quad y = \frac{t^2}{1+t}.$$

Determine the Cartesian equation of C. **[5 marks]**

Total 25 marks

SECTION C (MODULE 3)

Answer BOTH questions.

5. (a) Show that $\lim\limits_{h \to 0} \dfrac{h}{\sqrt{x+h} - \sqrt{x}} = 2\sqrt{x}$. **[6 marks]**

(b) The function f (x) is such that $\mathrm{f}^{11}(x) = 18x + 4$. Given that f (2) = 14 and f (3) = 74, find the value of f (4).

[8 marks]

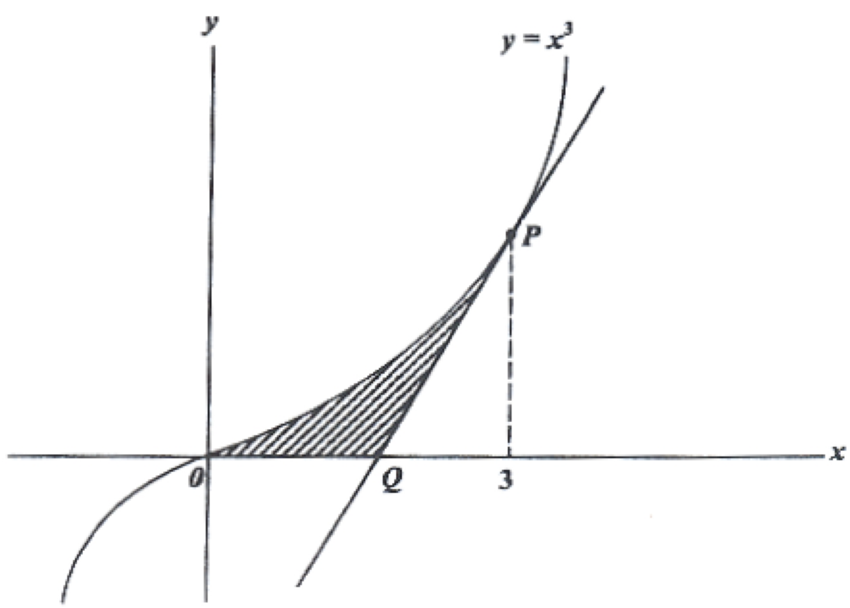
(c) If $y = \dfrac{x}{1+x^2}$, show that

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{(1+x^2)^2} - y^2$ **[5 marks]**

(ii) $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{2y(x^2 - 3)}{(1+x^2)^2}$ **[6 marks]**

Total 25 marks

6. (a) The diagram below, **not drawn to scale** is a sketch of the curve $y = x^3$ and the tangent PQ to the curve at $P(3, 27)$.



(i) Find

a) the equation of the tangent PQ **[4 marks]**

b) the coordinates of Q. **[1 mark]**

6. (a)

(ii) Calculate

a) the area of the shaded region, **[5 marks]**

b) the volume of the solid generated when the shaded region is rotated completely about the x-axis, giving your answer in terms of π.

[5 marks]

[If needed, the volume, V ,of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.]

(b) The gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - 8x + 5.$$

The curves passes through the point (0, 3).

(i) Find the equation of the curve. **[3 marks]**

(ii) Find the coordinates of the two stationary points of the curve in (b) (i) above and distinguish the nature of **each point.**

[7 marks]

Total 25 marks

END OF TEST