FORM TP 2005253

TEST CODE **02134020**

MAY/JUNE 2005

CARIBBEAN EXAMINATIONS COUNCIL

# ADVANCED PROFICIENCY EXAMINATION

## PURE MATHEMATICS

## UNIT 1 – PAPER 02

*2 hours*

25 MAY 2005 (p.m.)

This examination paper consists of THREE sections: Module1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

### INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

**Section A (Module 1)**

**Answer BOTH questions.**

1. (a) (i) Complete the table below for the function $|f(x)|$, where $f(x) = x(2 - x)$.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|
| $\lvert f(x) \rvert$ | 8 | | | | 0 | | 8 |

**[ 2 marks]**

(ii) Sketch the graph of $|f(x)|$ for $-2 \leq x \leq 4$. **[ 4 marks]**

(b) Find the value(s) of the real number, $k$, for which the equation $k(x^2 + 5) = 6 + 12x - x^2$ has equal roots. **[ 6 marks]**

(c) (i) If $2^{(x^2)} = 16^{(x-1)}$, find $x$. **[ 4 marks]**

(ii) Without using calculators or tables, evaluate

$(\sqrt{2} + 1)^3 - (\sqrt{2} - 1)^3$. **[ 4 marks]**

**Total 20 marks**



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2. (a) Prove, by Mathematical Induction, that $10^n - 1$ is divisible by 9 for all positive integers $n$. **[ 9 marks]**

(b) A pair of simultaneous equations is given by

$$px + 2y = 8$$
$$-4x + p^2y = 16$$

where $p \in \mathbf{R}$.

(i) Find the value of $p$ for which the system has an infinite number of solutions. **[ 3 marks]**

(ii) Find the solutions for this value of $p$. **[ 3 marks]**

(c) Find the **set of real values** of $x$ for which $\dfrac{x+4}{x-2} > 5$. **[ 5 marks]**

**Total 20 marks**

**Section B (Module 2)**

**Answer BOTH questions.**

3. The equation of the circle, $Q$, with centre $O$ is $x^2 + y^2 - 2x + 2y = 23$.

(a) Express the equation of $Q$ in the form $(x - a)^2 + (y - b)^2 = c$. **[ 5 marks]**

(b) Hence, or otherwise, state

(i) the coordinates of the centre of $Q$ **[ 2 marks]**

(ii) the radius of $Q$. **[ 1 mark ]**

(c) Show that the point $A(4, 3)$ lies on $Q$. **[ 3 marks]**

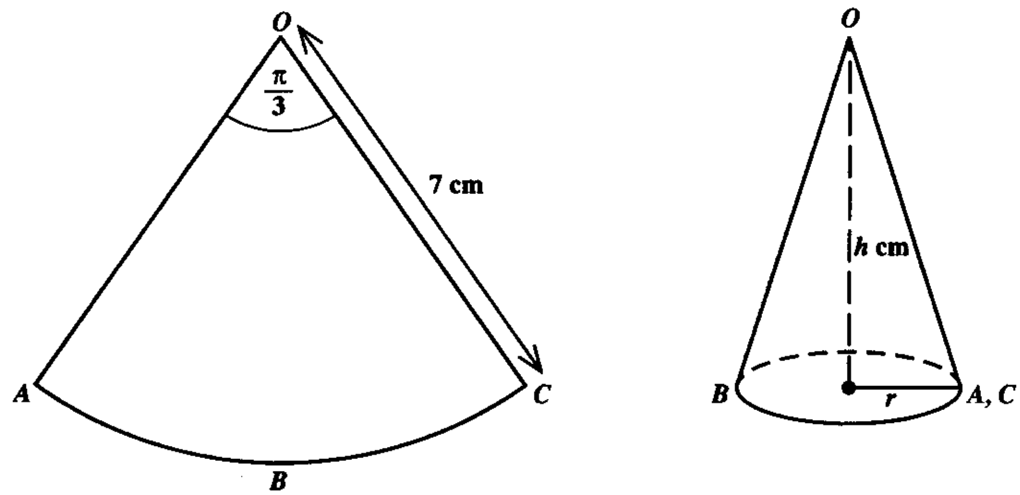
(d) Find the equation of the tangent to $Q$ at the point $A$. **[ 5 marks]**

(e) The centre of $Q$ is the midpoint of its diameter $AB$. Find the coordinates of $B$. **[ 4 marks]**

**Total 20 marks**

4. The diagrams shown below, **not drawn to scale**, represent

- a sector, $OABC$, of a circle with centre at $O$ and a radius of 7 cm, where angle $AOC$ measures $\frac{\pi}{3}$ radians.
- a right circular cone with vertex $O$ and a circular base of radius $r$ cm which is formed when the sector $OABC$ is folded so that $OA$ coincides with $OC$.




(a) (i) Express the arc length $ABC$ in terms of $\pi$. **[ 1 mark ]**

(ii) Hence, show that

a) $r = \frac{7}{6}$ **[ 3 marks]**

b) if $h$ cm is the height of the cone, then the exact value of $h$ is $\frac{7\sqrt{35}}{6}$.

**[ 2 marks]**

(b) (i) Show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. **[ 5 marks]**

(ii) The position vectors of two points $A$ and $B$ relative to the origin $O$ are

$$\boldsymbol{a} = 4\cos^2\theta\,\boldsymbol{i} + (6\cos\theta - 1)\,\boldsymbol{j}$$

$$\boldsymbol{b} = 2\cos\theta\,\boldsymbol{i} - \boldsymbol{j}.$$

By using the identity in (b) (i) above, find the value of $\theta$, $0 \le \theta \le \frac{\pi}{4}$, such that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular. **[ 5 marks]**

(c) Find the modulus of the complex number $z = \frac{25\,(2 + 3i)}{4 + 3i}$.

**[ 4 marks]**

**Total 20 marks**

## Section C (Module 3)

**Answer BOTH questions.**

**5.** (a) (i) State the value of $\lim\limits_{u \to 0} \dfrac{\sin u}{u}$. [ 1 mark ]

(ii) By means of the substitution $u = 3x$, show that $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x} = 3$.

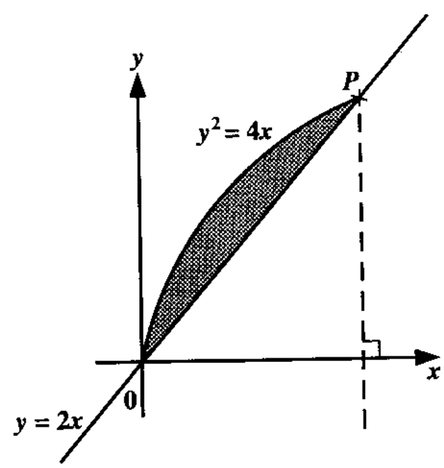
[ 4 marks]

(iii) Hence, evaluate $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 5x}$. [ 4 marks]

(b) If $y = \dfrac{A}{x} + Bx$, where A and B are constants, show that

$x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = y.$ [ 4 marks]

(c) The diagram below, **not drawn to scale**, shows part of the curve $y^2 = 4x$. $P$ is the point on the curve at which the line $y = 2x$ cuts the curve.




Find

(i) the coordinates of $P$ [ 3 marks]

(ii) the volume of the solid generated by rotating the shaded area through $2\pi$ radians about the $x$-axis. [ 4 marks]

**Total 20 marks**

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**6.** (a) Differentiate, with respect to $x$,

$(x^2 + 7)^5 + \sin 3x$. **[ 6 marks]**

(b) Determine the values of $x$ for which the function $y = x^3 - 9x^2 + 15x + 4$

(i) has stationary points **[ 3 marks]**

(ii) is increasing **[ 2 marks]**

(iii) is decreasing. **[ 2 marks]**

(c) (i) Use the substitution $t = a - x$ to show that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$. **[ 4 marks]**

(ii) If $\int_0^4 f(x)\, dx = 12$, use the substitution $t = x - 1$ to evaluate $\int_1^5 3f(x-1)\, dx$. **[ 3 marks]**

**Total 20 marks**

**END OF TEST**