FORM TP 02134032/SPEC

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1

ALGERBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 03/B

1 *hour* 30 *minutes*

The examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.
The maximum mark for each Module is 20.
The maximum mark for this examination is 60.
This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **THREE** significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2010**
Mathematical instruments
Silent, non-programmable electronic calculator

SECTION A (MODULE 1)

Answer this question.

1. (a) **p** and **q** are two given propositions.

(i) State the converse of $\mathbf{p} \rightarrow \mathbf{q}$. **[1 mark]**

(ii) Show that the contrapositive of the inverse of $\mathbf{p} \rightarrow \mathbf{q}$ is the converse of $\mathbf{p} \rightarrow \mathbf{q}$.

[2 marks]

(b) $f(n) = 2^n + 6^n$

(i) Show that $f(k + 1) = 6f(k) - 4(2^k)$ **[3 marks]**

(ii) Hence, or otherwise, prove by mathematical induction that, for $n \in \mathbb{N}$, $f(n)$ is divisible by 8.

[4 marks]

(c) (i) On the same diagram, sketch the graph of $y = x + 2$ and the graph of $y = \left|\frac{1}{x-2}\right|$, showing clearly on your sketch the coordinates of any points at which the graphs cross the axes, and state the equations of any asymptotes.

[6 marks]

(c) (ii) Find the range of values of x for which

$$x + 2 < \left|\frac{1}{x - 2}\right|.$$

[4 marks]

Total 20 marks

SECTION B (MODULE 2)

Answer this question

2. (a) (i) Using $\sin^2\theta + \cos^2\theta \equiv 1$ show that $\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$.

[2 marks]

(ii) Hence, or otherwise, prove that $\operatorname{cosec}^4\theta - \cot^4\theta \equiv \operatorname{cosec}^2\theta + \cot^2\theta$.

[2 marks]

(b) A curve C has parametric equations

$$x = \sin^2\theta,\ y = 2\tan\theta,\ 0 \le \theta < 90^0.$$

Find the Cartesian equation of C. **[4 marks]**

(c) The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(i) the coordinates of C, **[3 marks]**

(ii) the angle between l_1 and l_2, correct to 2 decimal places.

[4 marks]

(iii) Show that the vector $\mathbf{n} = 4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}$ is perpendicular to l_1 and l_2.

[2 marks]

(iv) Hence find the vector equation of the plane, $\mathbf{r} \cdot \mathbf{n} = d$, through the point $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$.

[3 marks]

Total 20 marks

SECTION C (MODULE 3)

Answer this question

3. (a) $S_n = 1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n + 1)! - 1$.

Show that $\lim_{n \to \infty} \frac{S_n}{S_{n+1}} = 0$. **[4 marks]**

(b) Using $\lim_{x \to 0} \frac{\sin x}{x} = 1$, differentiate from first principles $f(x) = \cos x$.

[5 marks]

(c) A circular patch of oil on the surface of a pool of water has radius r metres at time t hours after spillage occurs. At time 2:00 p. m., one hour after the spillage, the radius of the patch of oil is 5 metres. In a model, the rate of increase of r is taken to be proportional to $\frac{1}{r}$.

(i) Form a differential equation for r in terms of t, involving a constant of proportionality k

[1 mark]

(ii) Solve this differential equation and hence show that the radius of the patch of oil is proportional to the square root of the time elapsed since the spillage.

[7 marks]

(iii) Determine the time, to the nearest minute, at which the model predicts that the radius of the patch of oil will be 12 metres.

[3 marks]

Total 20 marks

END OF TEST