



CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 01

2 hours

18 MAY 2007 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2, and Module 3.

Each section consists of 5 questions.

The maximum mark for each section is 40.

The maximum mark for this examination is 120.

This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables

Electronic calculator

Graph paper

Section A (Module 1)

Answer ALL questions.

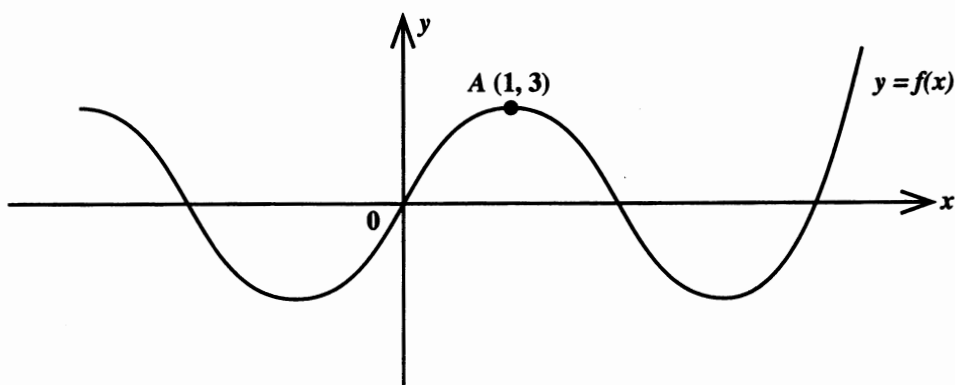
1. Given that $x - 1$ is a factor of the function $f(x) = x^3 + px^2 - x - 2$, $p \in \mathbf{R}$, find
- (a) the value of p [2 marks]
- (b) the remaining factors of $f(x)$. [4 marks]

Total 6 marks

2. (a) Solve, for x , the equation $(3^x)^2 = 27^{(x-2)}$. [4 marks]
- (b) Express $\frac{5 - \sqrt{3}}{2 + \sqrt{3}}$ in the form $x + y\sqrt{3}$ where $x, y \in \mathbf{Z}$. [5 marks]

Total 9 marks

3. (a) The diagram below (**not drawn to scale**) shows the graph $y = f(x)$ which has a local maximum point at $A(1, 3)$.



Determine the coordinates of the maximum point on each of the following graphs.

- (i) $y = f(x) - 2$ [2 marks]
- (ii) $y = f(x + 3)$. [2 marks]
- (b) The function, f , is defined on \mathbf{R} by $f: x \rightarrow 3x - 2$.
- (i) Show that f is one-to-one. [2 marks]
- (ii) Hence, or otherwise, find the value of $x \in \mathbf{R}$ for which $f(f(x + 3)) = f(x - 3)$. [4 marks]

Total 10 marks

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4. (a) Solve $|x - 4| - 6 > 0$ for all $x \in \mathbf{R}$. [5 marks]

- (b) Find the real numbers u , v and w such that $-3x^2 - x + 2 \equiv u(x + v)^2 + w$. [3 marks]

Total 8 marks

5. Solve the following pair of simultaneous equations.

$$\begin{aligned}x^2 + xy &= 2 \\ y + 3x &= 5\end{aligned}$$

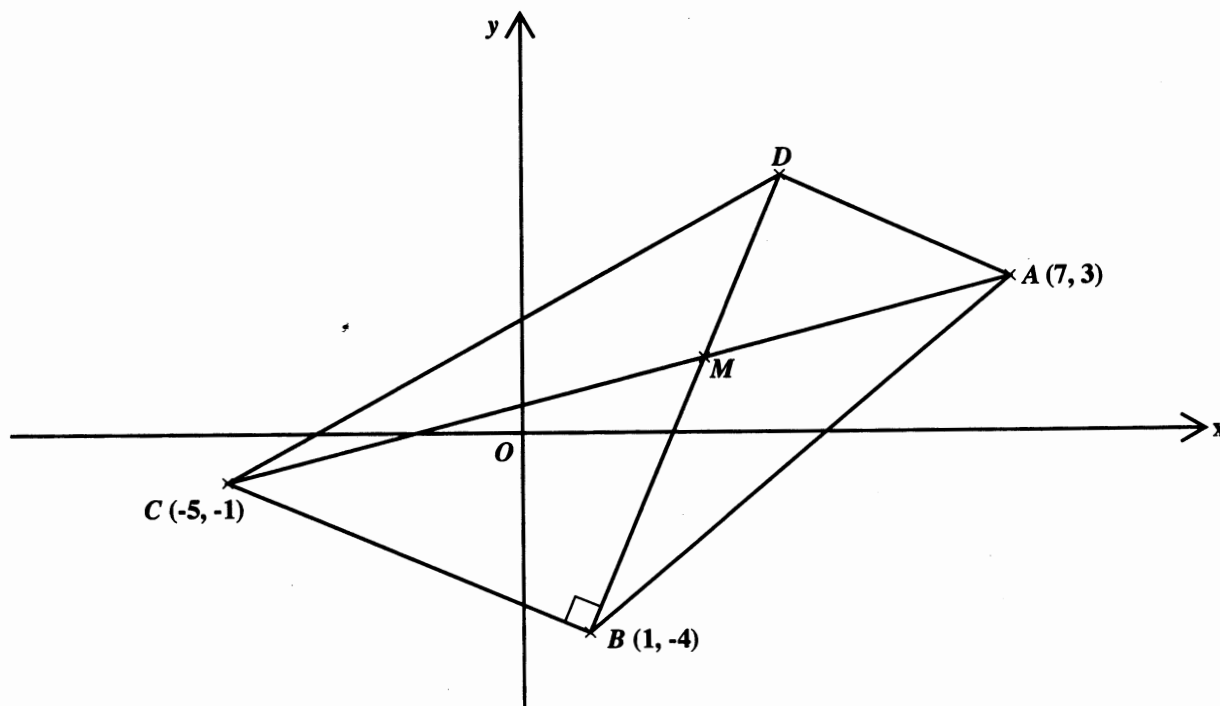
[7 marks]

Total 7 marks

Section B (Module 2)

Answer ALL questions.

6. In the diagram below (not drawn to scale), the points $A(7, 3)$, $B(1, -4)$, $C(-5, -1)$ are three vertices of a quadrilateral $ABCD$. The line BD is perpendicular to BC and M is the point of intersection of the lines AC and BD .



- (a) Find the equation of
- (i) the line AC [3 marks]
 - (ii) the line BD . [3 marks]
- (b) Hence, find the coordinates of M . [3 marks]

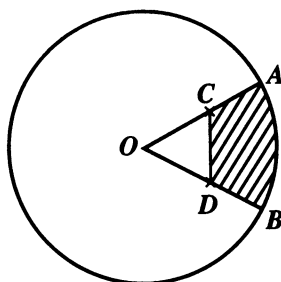
Total 9 marks

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7. (a) Express $\cos \theta - \sin \theta$ in the form $R \cos (\theta + \alpha)$ where $R, \alpha \in \mathbf{R}$, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [5 marks]
- (b) Hence, find the general solution of $\cos \theta - \sin \theta = 1$. [3 marks]

Total 8 marks

8. The circle in the diagram below, **not drawn to scale**, has centre O and the acute angle $AOB = \frac{\pi}{6}$ radians. $OA = 6$ cm and C and D are the midpoints of OA and OB respectively.



Express in terms of π

- (a) the length of arc AB [2 marks]
- (b) the area of the shaded region $ABDC$. [4 marks]

Total 6 marks

9. (a) Given $z = 4 + 3i$, express $\frac{\bar{z}}{z}$ in the form $a + bi$ where $a, b \in \mathbf{R}$. [5 marks]
- (b) Find $\left| \frac{\bar{z}}{z} \right|$. [2 marks]

Total 7 marks

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10. (a) The position vectors of points A and B with respect to an origin O are given by $\vec{OA} = 3\mathbf{i} + 2\mathbf{j}$ and $\vec{OB} = 2\mathbf{i} - 4\mathbf{j}$. Find
- (i) \vec{AB} in terms of \mathbf{i} and \mathbf{j} [2 marks]
 - (ii) the magnitude of \vec{AB} [2 marks]
 - (iii) the position vector of the point M that divides AB internally in the ratio 1:2. [3 marks]
- (b) Determine whether \vec{OA} is perpendicular to \vec{OB} . [3 marks]

Total 10 marks

Section C (Module 3)

Answer ALL questions.

11. (a) Determine $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x^3 - 4x}$. [4 marks]

- (b) Obtain the real values of x such that the function

$$f(x) = \frac{x^2 + 1}{(|2x - 3| - 9)}$$

is continuous.

[4 marks]

Total 8 marks

12. (a) Differentiate, with respect to x , the function $f(x) = \frac{x^2 - 4}{x^3 + 1}$. [4 marks]

- (b) Using the substitution $u = \sin 2x$, or otherwise, evaluate $\int_0^{\pi/4} \sin 2x \cos 2x \, dx$.

[4 marks]

Total 8 marks

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13. The curve $y = px^3 + qx + r$ passes through the origin O and the point $P(1, 2)$. The gradient of the curve at P is equal to 8.

(a) Calculate the values of the constants p , q and r . [6 marks]

(b) Obtain the equation of the normal to the curve at P . [2 marks]

Total 8 marks

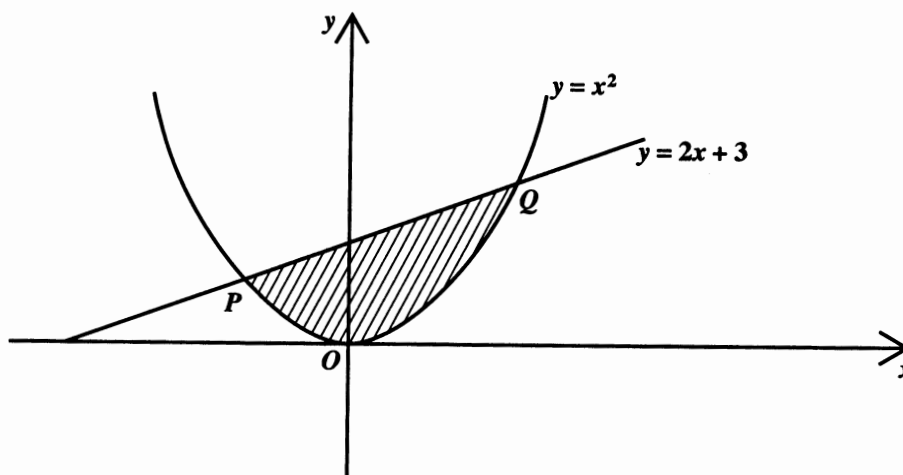
14. For the function $f: x \rightarrow 12x - x^3$, determine

(a) the stationary points [4 marks]

(b) the nature of EACH of the stationary points. [3 marks]

Total 7 marks

15. In the diagram below (**not drawn to scale**), the line $y = 2x + 3$ cuts the curve $y = x^2$ at the points P and Q .



(a) Determine the coordinates of P and Q . [4 marks]

(b) Calculate the area of the shaded portion POQ shown in the diagram above. [5 marks]

Total 9 marks

END OF TEST