CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

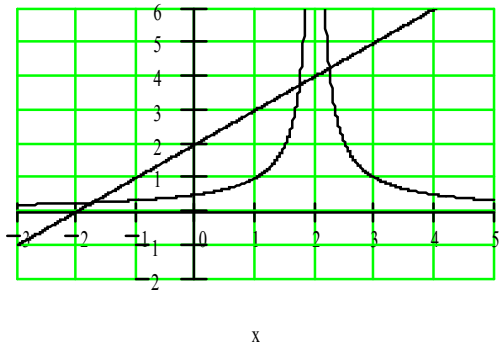
UNIT 1

ALGEBRA, GEOMETRY AND CALCULUS

SPECIMEN PAPER

PAPER 03/B

SOLUTIONS AND MARK SCHEMES

Question	Details	Marks
1 (a) (i)	The converse of $\mathbf{p} \rightarrow \mathbf{q}$ is $\mathbf{q} \rightarrow \mathbf{p}$	1
(ii)	The inverse of $\mathbf{p} \rightarrow \mathbf{q}$ is $\sim \mathbf{p} \rightarrow \sim \mathbf{q}$	1
	The contrapositive of $\sim \mathbf{p} \rightarrow \sim \mathbf{q}$ is $\sim\sim \mathbf{q} \rightarrow \sim\sim \mathbf{p} = \mathbf{q} \rightarrow \mathbf{p}$	1
(b) (i)	$f(k+1) = 2^{k+1} + 6^{k+1}$	1
	$= 2(2^k) + 6(6^k)$	1
	$= 6(2^k + 6^k) - 4(2^k)$	
	$= 6f(k) - 4(2^k)$	1
(ii)	Assume $f(k)$ is divisible by 8 $\quad f(1) = 2 + 6 = 8$ (true)	1
	$f(k+1) = 6f(k) - 2 \times 2(2^k) = 6f(k) - 2(2^{k+1})$	1
	$= 6f(k) - 8\left(\frac{1}{4}2^{k+1}\right)$ which is divisible by 8	1
	Since $f(k)$ is true for $k = 1$ and true for $f(k+1)$ then true for all $n \in \mathbb{N}$	1
(c) (i)	$\dfrac{x+2}{\dfrac{1}{\lvert x-2 \rvert}}$ 	
	line , positive gradient, intercepts (-2, 0), (0, 2)	1
	curve…..branch > 2	1
	curve…...branch < 2	1
	intercept (0, ½)	1
	asymptotes: $x = 2, y = 0$	1
	(1) (1)	1
(ii)	$x + 2 = \dfrac{1}{x-2} \quad x^2 = 5 \quad x = \sqrt{5} \quad (-\sqrt{5}$ not applicable)	1
	$x + 2 = \dfrac{1}{2-x} \quad x^2 = 3 \quad x = \sqrt{3} \quad (-\sqrt{3}$ not applicable)	1
	$\sqrt{3} < x < \sqrt{5}$	1
	(1) (1)	1
		20
S. O. (A) 3, (B) 5, (C) 5, (F) 3		

Question	Details	Marks
2 (a) (i)	Dividing throughout by $\sin^2\theta$ gives $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$	1
	$1 + \cot^2\theta = \text{cosec}^2\theta$ $\quad \text{cosec}^2\theta - \cot^2\theta = 1$	1
(ii)	$\text{cosec}^4\theta - \cot^4\theta = (\text{cosec}^2\theta - \cot^2\theta)(\text{cosec}^2\theta + \cot^2\theta)$	1
	$= \text{cosec}^2\theta + \cot^2\theta$	1
(b)	$\frac{1}{x} = \text{cosec}^2\theta \quad \frac{2}{y} = \cot\theta$	1
	(1) (1)	1
	From (a) (i) $\frac{1}{x} - \frac{4}{y^2} = 1$	1
	$y^2 - 4x = xy^2$	1
(c) (i)	Comparing components of **j**: $3 + 2\lambda = 9 \quad \lambda = 3$	1
	Comparing components of **i**: $2 + 3 = 5\mu \quad \mu = 1$	1
	C (5, 9, –1)	1
(ii)	$\cos\theta = \frac{(\mathbf{i} + 2\mathbf{j} + \mathbf{k}).(5\mathbf{i} + 2\mathbf{k})}{\sqrt{6} \times \sqrt{29}}$ (1 mark each for numerator and denominator)	1
		1
	$\theta = \cos^{-1}\left(\frac{7}{\sqrt{174}}\right)$	1
	$= 57.95^0$	1
(iii)	$(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) \cdot (5\mathbf{i} + 2\mathbf{k}) = 0$	1
	$(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = 0$	1
	$\mathbf{r}.(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) \cdot (4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k})$	1
	(1) (1)	1
	$\mathbf{r}.(4\mathbf{i} + 3\mathbf{j} - 10\mathbf{k}) = 57$	1
		20

S. O. (A) 5 (ii), 6 (B) 5, (C) 7, 9, 10

Question	Details	Marks
3 (a)	$\lim_{n\to\infty}\frac{S_n}{S_{n+1}} = \lim_{n\to\infty}\frac{(n+1)!-1}{(n+2)!-1}$	1
	$= \frac{\lim_{n\to\infty}(n+1)!-1}{\lim_{n\to\infty}(n+2)!-1} = \frac{\lim_{n\to\infty}(n+1)!-1}{\lim_{n\to\infty}(n+2)(n+1)!-1}$ (1 mark each for limit)	1 1
	$= \lim_{n\to\infty}\frac{1}{n+2}$	1
	$= 0$	1
(b)	$\frac{\mathrm{d}}{\mathrm{d}x}\cos x = \lim_{\delta x\to 0}\frac{\cos(x+\delta x)-\cos x}{\delta x}$	1
	$= \lim_{\delta x\to 0}\frac{-2\sin\left(\frac{x+\delta x+x}{2}\right)\sin\left(\frac{x+\delta x-x}{2}\right)}{\delta x}$	1
	$= \lim_{\delta x\to 0}\frac{-2\sin\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$	1
	Given $\lim_{\delta x\to 0}\frac{\sin x}{x} = 1$ $\quad \frac{1}{2}\lim_{\delta x\to 0}\frac{\sin\frac{\delta x}{2}}{\left(\frac{\delta x}{2}\right)} = \frac{1}{2}$	1
	$\lim_{\delta x\to 0}\frac{-2\sin\left(x+\frac{\delta x}{2}\right)}{2} = -\sin x$	1
(c) (i)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r}$	1
(ii)	$\int r\,\mathrm{d}r = \int k\,\mathrm{d}t$	1
	$\frac{1}{2}r^2 = kt + A$	1
	(1) (1)	1
	$r = 0$ $t = 0$ gives $A = 0$	1
	$r = 5$ $t = 1$ gives $k = \frac{25}{2}$	1
	$r^2 = 25t$ $\quad r = 5\sqrt{t}$	1

Question	Details	Marks
3 (c) (iii)	$12 = 5\sqrt{t}$ (1) $\quad t = \left(\frac{12}{5}\right)^2 = 5.76\text{h} = 5\text{ hrs }46\text{ mins}$ (1) Time when radius is 12 metres is 6:46 p. m	1 1 1 20
	S. O. (A) 3, 4, 5, (B) 2 (iv), (C) 9 (i) (ii)	