

**CARIBBEAN EXAMINATIONS COUNCIL**
ADVANCED PROFICIENCY EXAMINATION**PURE MATHEMATICS****UNIT 1 – PAPER 03/B****ALGEBRA, GEOMETRY AND CALCULUS***1 ½ hours***16 MAY 2008 (p.m.)**

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 3 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2008**

Mathematical instruments

Silent, non-programmable, electronic calculator

SECTION A (Module 1)

Answer this question.

1. (a) One root of the quadratic equation $x^2 + 12x + k = 0$ is three times the other and k is a constant.

Find

(i) the roots of the equation [3 marks]

(ii) the value of k . [2 marks]

- (b) (i) The function $f(x)$ has the property that

$$f(2x + 3) = 2f(x) + 3, x \in \mathbf{R}.$$

If $f(0) = 6$, find the values of $f(3)$ and $f(9)$. [4 marks]

(ii) Solve for x the equation $5^x - 5^{x-2} = 15\,000$. [4 marks]

- (c) A computer manufacturer finds that when x million dollars are spent on research, the profit, $P(x)$, in millions of dollars, is given by

$$P(x) = 20 + 5 \log_3 (x + 3).$$

(i) What is the profit if 6 million dollars are spent on research? [3 marks]

(ii) How much should be spent on research to make a profit of 40 million dollars? [4 marks]

Total 20 marks

SECTION B (Module 2)

Answer this question.

2. (a) Find the values of x in the range $0 \leq x \leq 2\pi$ such that

$$4 \cos^3 x + 2 \cos x - 5 \sin 2x = 0. \quad [10 \text{ marks}]$$

- (b) (i) Determine the value of the real number t such that the vectors $\mathbf{p} = 4\mathbf{i} + 5\mathbf{j}$ and $\mathbf{q} = 3\mathbf{i} - t\mathbf{j}$ are perpendicular. [2 marks]

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- (ii) Given that vectors $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = \mathbf{i} + 5\mathbf{j}$, find the acute angle θ between \mathbf{u} and \mathbf{v} . [4 marks]
- (iii) Given that the vector \mathbf{u} in (b) (ii) above represents a force F with respect to the origin O , and axes Ox and Oy , calculate
- a) the magnitude of F [2 marks]
- b) the angle ϕ of inclination of F to Ox . [2 marks]

Total 20 marks

SECTION C (Module 3)

Answer this question.

3. (a) A curve has equation $y = x + \frac{4}{x}$.
- (i) Show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} = y$. [5 marks]
- (ii) Find the equation of the normal to the curve at the point where $x = 4$. [5 marks]
- (b) Find $\int \frac{x^2 + x - 1}{x^4} dx$. [4 marks]
- (c) The volume of the liquid in a container is $V \text{ cm}^3$. The liquid leaks from the container at the rate of $30t \text{ cm}^3$ per sec, where t is the time in seconds.
- (i) Write down a differential equation for V with respect to time t secs. [2 marks]
- (ii) Find the amount of liquid lost in the 3rd second. [4 marks]

Total 20 marks

END OF TEST