FORM TP 2004245

TEST CODE **02134010**

MAY/JUNE 2004

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

MATHEMATICS

UNIT 1 – PAPER 01

$1\frac{1}{2}$ hours

21 MAY 2004 (p.m.)

This examination paper consists of THREE sections: Module 1.1, Module 1.2, and Module 1.3.

Each section consists of 5 questions.
The maximum mark for each section is 30.
The maximum mark for this examination is 90.
This examination paper consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.

2. Answer **ALL** questions from the **THREE** sections.

3. Unless otherwise stated in the question, all numerical answers **MUST** be given exactly **OR** to three significant figures as appropriate.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

SECTION A (MODULE 1.1)

Answer ALL questions.

1. The function $f(x) = x^3 - p^2x^2 + 2x - p$ has remainder -5 when it is divided by $x + 1$. Find the possible values of the constant p. **[6 marks]**

2. (a) Given that $x > y$, and $k < 0$ for the real numbers x, y and k, show that $kx < ky$. **[4 marks]**

(b) Solve, for $x \in \mathbf{R}$, the equation

$$x^2 - 6\,|\,x\,| + 8 = 0.$$ **[4 marks]**

3. (a) Show that $\frac{4}{2^x} = 2^{2-x}$. **[1 mark]**

(b) Solve, for x, the equation

$$2^x + 2^{2-x} = 5.$$ **[4 marks]**

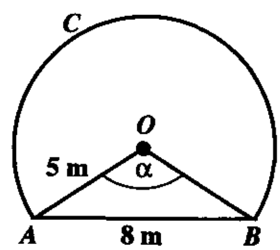
4. The functions f and g are defined on $\mathbf{R}$ by

$$f : x \to -3x + 6, \quad g : x \to x + 7.$$

Solve, for x, the equation

$$f(g(2x + 1)) = 30.$$ **[5 marks]**

5. The figure below **(not drawn to scale)** represents a cross-section through a tunnel. The cross-section is part of a circle with radius 5 metres and centre O. The width AB of the floor of the tunnel is 8 metres.



Calculate

(a) the size, in radians, of the angle α **[3 marks]**

(b) the length of the arc ACB. **[3 marks]**

Total 30 marks

SECTION B (MODULE 1.2)

Answer ALL questions.

6. Obtain the Cartesian equation of the curve whose parametric representation is $x = 2t^2 + 3$, $y = 3t^4 + 2$ in the form $y = Ax^2 + Bx + C$, where A, B and C are real numbers.
[6 marks]

7. Find the range of values of $x \in \mathbf{R}$ for which $\frac{x-2}{x+3} > 0, x \neq -3$. **[6 marks]**

8. (a) Show that $\frac{\sin 2A}{1 - \cos 2A} = \cot A$, for $\cos 2A \neq 1$. **[3 marks]**

(b) Solve the equation $\cos 2\theta = 3 \cos \theta - 2$ for $0 \leq \theta \leq \pi$. **[4 marks]**

9. Given that α and β are the roots of the equation $x^2 - 3x - 1 = 0$, find the equation whose roots are $1 + \alpha$ and $1 + \beta$. **[5 marks]**

10. The position vector of a point P is $\mathbf{i} + 3\mathbf{j}$. Find

(a) the unit vector in the direction of $\overrightarrow{OP}$ **[2 marks]**

(b) the position vector of a point Q on $\overrightarrow{OP}$ produced such that $|\overrightarrow{OQ}| = 5$ **[2 marks]**

(c) the value of t such that the vector $3t\,\mathbf{i} + 4\mathbf{j}$ is perpendicular to the vector $\overrightarrow{OP}$. **[2 marks]**

Total 30 marks

SECTION C (MODULE 1.3)

Answer ALL questions.

11. (a) Given that $\lim_{x \to -2} \{4f(x)\} = 5$,

evaluate $\lim_{x \to -2} \{f(x) + 2x\}$. **[5 marks]**

12. Differentiate **from first principles** the function

$$f(x) = x^3,$$

with respect to x. **[6 marks]**

13. Given that $f(x) = rx^2 + sx + t,\ r \neq 0$,

(a) find (i) $f'(x)$

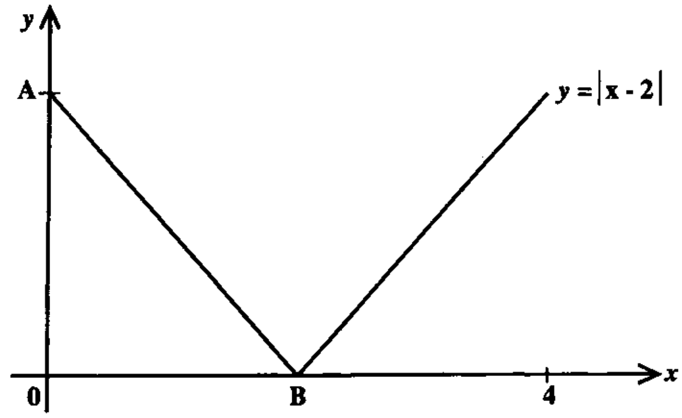
(ii) $f''(x)$ **[2 marks]**

(b) find, in terms of r and s, the conditions under which $f(x)$ will have a maximum **[3 marks]**

(c) find the maximum. **[3 marks]**

14. The curve $y = px^3 + qx^2 + 3x + 2$ passes through the point $T\,(1, 2)$ and its gradient at T is 7. Find the values of the constants p and q. **[5 marks]**

15. The diagram below is a rough diagram of $y = | x - 2 |$ for real values of x from $x = 0$ to $x = 4$.



(a) Find the coordinates of the points A and B. **[2 marks]**

(b) Find the volume generated by rotating the triangle OAB shown above through 360° **about the** x**-axis**. **[4 marks]**

Total 30 marks

END OF TEST