



CARIBBEAN EXAMINATIONS COUNCIL
ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 03/B

ALGEBRA, GEOMETRY AND CALCULUS

1 ½ hours

09 JUNE 2010 (p.m.)

This examination paper consists of **THREE** sections: Module 1, Module 2 and Module 3.

Each section consists of 1 question.

The maximum mark for each Module is 20.

The maximum mark for this examination is 60.

This examination consists of 4 printed pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Write your solutions, with full working, in the answer booklet provided.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)

Mathematical formulae and tables (provided) – **Revised 2009**

Mathematical instruments

Silent, non-programmable, electronic calculator



SECTION A (Module 1)

Answer this questions.

1. (a) The roots of the cubic equation $x^3 + px^2 + qx + 48 = 0$ are α , 2α and 3α . Find
- (i) the value of α [2 marks]
 - (ii) the values of the constants p and q . [4 marks]
- (b) The function f on \mathbb{R} is given by
- $$f: x \rightarrow 3x - 2.$$
- (i) Show that f is one-to-one. [3 marks]
 - (ii) Find the value of x for which
- $$f(f(x)) = f(x + 3).$$
- [4 marks]
- (c) Prove by mathematical induction that
- $$9^n - 1 \text{ is divisible by } 8$$
- for all $n \in \mathbb{N}$. [7 marks]

Total 20 marks

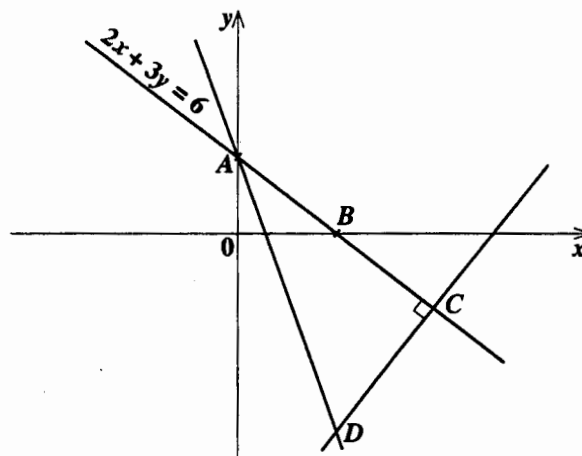
SECTION B (Module 2)

Answer this questions.

2. A surveyor models the boundaries and extent of a triangular plot of land on a Cartesian plane as shown in the diagram below (**not drawn to scale**). The line $2x + 3y = 6$ meets the y -axis at A and the x -axis at B .

C is the point on the line $2x + 3y = 6$ such that $AB = BC$.

CD is drawn perpendicular to AC to meet the line through A parallel to $5x + y = 7$ at D .



- (a) Find
- (i) the coordinates of A , B and C [6 marks]
 - (ii) the equations of the lines CD and AD . [5 marks]
- (b) Show that the point D has coordinates $(2, -8)$. [4 marks]
- (c) Calculate the area of triangle ACD . [5 marks]

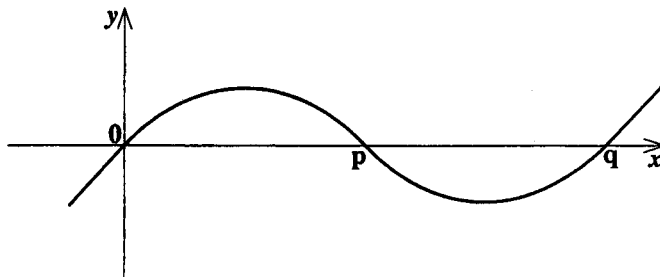
Total 20 marks

SECTION C (Module 3)

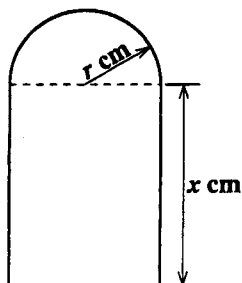
Answer this questions.

3. (a) Find $\int (\cos 5x + \tan^2 x) dx$. [4 marks]

- (b) Part of the curve $y = x(x-1)(x-2)$ is shown in the figure below (not drawn to scale).



- (i) Find the values of p and q . [2 marks]
- (ii) Hence find the area of the region enclosed by the curve and the x -axis from $x = 0$ to $x = q$. [5 marks]
- (c) A piece of wire, 60 cm long, is bent to form the shape shown in the figure below, (not drawn to scale). The shape consists of a semi-circular arc of radius r cm and three sides of a rectangle of height x cm.



- (i) Express x in terms of r . [3 marks]
- (ii) Show that the enclosed area A cm² is given by

$$A = 60r - 2r^2 \left[1 + \frac{\pi}{4} \right].$$
 [3 marks]

- (iii) Find the exact value of r for the stationary point of A . [3 marks]

Total 20 marks

END OF TEST