FORM TP 2006257

TEST CODE **02134010**

MAY/JUNE 2006

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 01

2 hours

19 MAY 2006 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2, and Module 3.

Each section consists of 5 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination paper consists of 7 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer ALL questions.

1. (a) The function $f(x)$ is given by $f(x) = x^4 - (p + 1)x^2 + p$, $p \in \mathbf{N}$.

(i) Show that $(x - 1)$ is a factor of $f(x)$ for all values of p. [2 marks]

(ii) If $(x - 2)$ is a factor of $f(x)$, find the value of p. [2 marks]

(b) Given that $\sum_{r=1}^{n} r = \frac{n}{2}(n + 1)$, show that $\sum_{r=1}^{n} (3r + 1) = \frac{1}{2} n(3n + 5)$. [4 marks]

Total 8 marks

2. (a) Let $A = \{x : 2 \leq x \leq 7\}$ and $B = \{x : |x - 4| \leq h\}$, $h \in \mathbf{R}$.

Find the **LARGEST** value of h for which $B \subset A$. [6 marks]

(b) Let $x, y, k \in \mathbf{R}$ such that $(x + \frac{1}{2} y)^2 + ky^2 \equiv x^2 + xy + y^2$.

Find the value of k. [3 marks]

Total 9 marks

3. (a) (i) Find $a, b \in \mathbf{R}$ such that $\frac{3x}{x+1} - 2 \equiv \frac{ax + b}{x + 1}$, where $x \neq -1$. [2 marks]

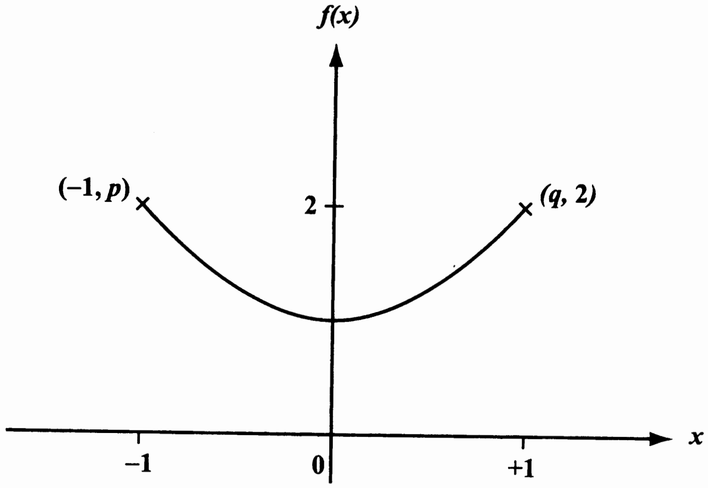
(ii) Hence, find the range of values of $x \in \mathbf{R}$ for which $\frac{3x}{x+1} > 2$. [4 marks]

(b) Without the use of calculators or tables, show that $\frac{4^2}{\sqrt{2} \text{ x } 8^{-1/3}} = 2^4 (\sqrt{2})$.

[4 marks]

Total 10 marks

4. The diagram below (**not drawn to scale**) represents the graph of the function $f(x) = x^2 + 1$, $-1 \leq x \leq 1$ and $p, q \in \mathbf{R}$.



(a) Find

(i) the value of p and of q [2 marks]

(ii) the range of the function $f(x)$ for the given domain. [1 mark]

(b) Determine whether $f(x)$

(i) is surjective (onto) [1 mark]

(ii) is injective (one-to-one) [1 mark]

(iii) has an inverse. [1 mark]

Total 6 marks

5. Find the values of $m, n \in \mathbf{R}$ for which the system of equations

$$x + 2y = 1$$
$$2x + my = n$$

(a) possesses a unique solution [3 marks]

(b) is inconsistent [2 marks]

(c) possesses infinitely many solutions. [2 marks]

Total 7 marks

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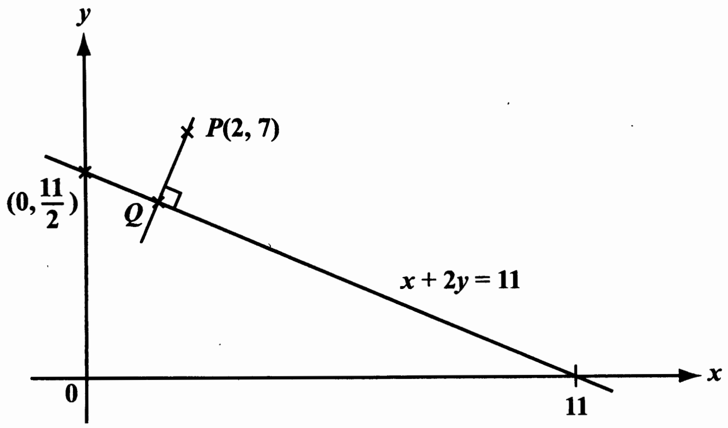
Section B (Module 2)

Answer ALL questions.

6. In the diagram below **(not drawn to scale)**, the straight line through the point $P(2, 7)$ and perpendicular to the line $x + 2y = 11$ intersects $x + 2y = 11$ at the point Q.



Find

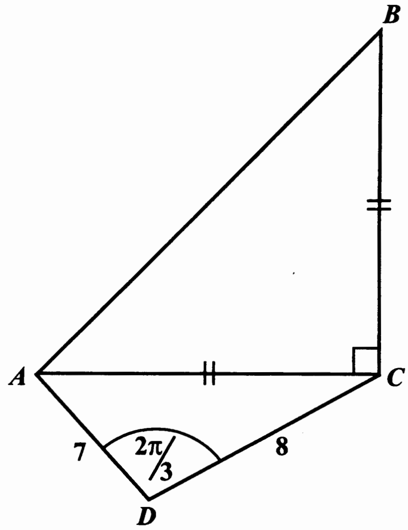
(a) the equation of the line through P and Q **[2 marks]**

(b) the coordinates of the point Q **[3 marks]**

(c) the EXACT length of the line segment PQ. **[2 marks]**

Total 7 marks

7. In the diagram below (**not drawn to scale**), $AC = BC$, $AD = 7$ units, $DC = 8$ units, angle $ACB = \frac{\pi}{2}$ radians and angle $ADC = \frac{2\pi}{3}$ radians.



Find the EXACT length of

(a) AC **[5 marks]**

(b) AB. **[3 marks]**

Total 8 marks

8. (a) Solve the equation $4\cos^2\theta - 4\sin\theta - 1 = 0$ for $0 \le \theta \le \pi$. **[5 marks]**

(b) Show that $\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x$. **[3 marks]**

Total 8 marks

9. (a) The roots of the quadratic equation $x^2 + 6x + k = 0$ are $-3 + 2i$ and $-3 - 2i$. Find the value of the constant k. **[2 marks]**

(b) Find the real numbers u and v such that $\frac{u+2i}{3-4i} \equiv 1 + vi$. **[6 marks]**

Total 8 marks

10. Given the vectors $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = 3\mathbf{i} - 2\mathbf{j}$,

(a) find $x, y \in \mathbf{R}$ such that $x\mathbf{p} + y\mathbf{q} = -3\mathbf{i} - 11\mathbf{j}$ **[7 marks]**

(b) show that $\mathbf{p}$ and $\mathbf{q}$ are perpendicular. **[2 marks]**

Total 9 marks

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Section C (Module 3)

Answer ALL questions.

11. (a) Find $\lim_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - 3x + 2}$. **[3 marks]**

(b) Find the values of $x \in \mathbf{R}$ such that the function

$$f(x) = \frac{9 - x^2}{(x^2 - 3)(|x| - 3)}$$

is discontinuous. **[4 marks]**

Total 7 marks

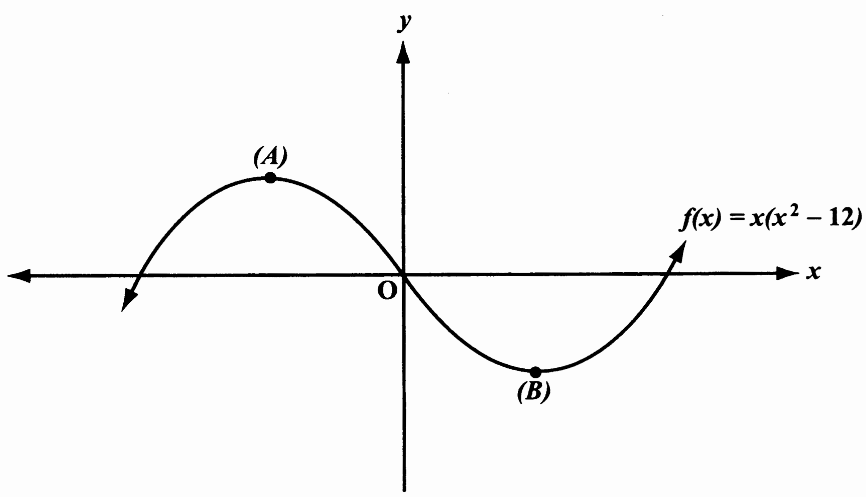
12. (a) The function $f(x)$ is defined by $f(x) = \dfrac{2 - x}{x^2}$ for $x \in \mathbf{R}$, $x \neq 0$.

Determine the nature of the critical value(s) of $f(x)$. **[6 marks]**

(b) Differentiate, with respect to x, $f(x) = \sin^2(x^2)$. **[3 marks]**

Total 9 marks

13. The diagram below (**not drawn to scale**) is a sketch of the section of the function $f(x) = x\,(x^2 - 12)$ which passes through the origin O. A and B are the stationary points on the curve.



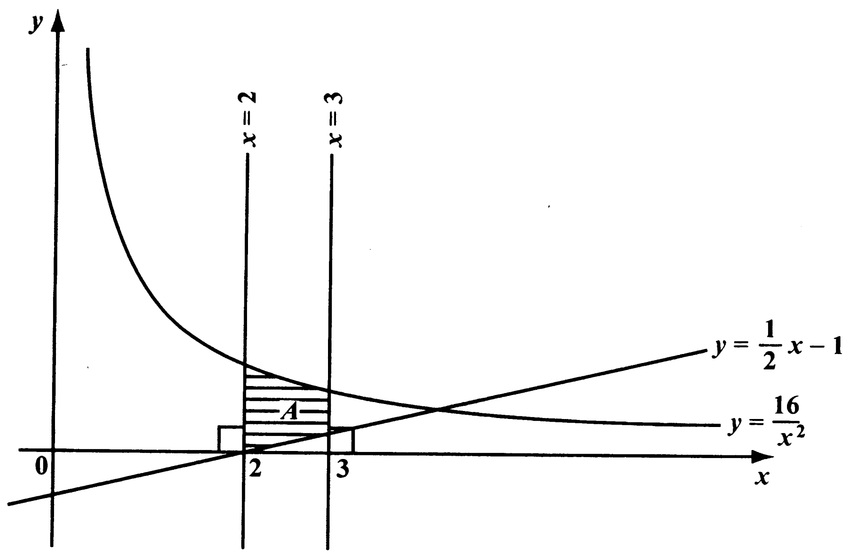
Find

(a) the coordinates of each of the stationary points, A and B **[5 marks]**

(b) the equation of the normal to the curve $f(x) = x\,(x^2 - 12)$ at the origin. **[4 marks]**

Total 9 marks

14. The diagram below (**not drawn to scale**) shows the shaded area, A, bounded by the curve $y = \frac{16}{x^2}$ and the lines $y = \frac{1}{2}x - 1$, $x = 2$ and $x = 3$.



(a) Express the shaded area, A, as the difference of two definite integrals. [1 mark]

(b) Hence, show that $A = 16\int_2^3 x^{-2}\,dx - \frac{1}{2}\int_2^3 x\,dx + \int_2^3 dx$. [2 marks]

(c) Find the value of A. [3 marks]

Total 6 marks

15. Use the result $\int_o^a f(x)dx = \int_o^a f(a-x)dx, a > 0$, to show that

(a) $\int_o^\pi x \sin x\,dx = \int_o^\pi (\pi - x)\sin x\,dx$. [2 marks]

(b) Hence, show that

(i) $\int_o^\pi x \sin x\,dx = \pi\int_o^\pi \sin x\,dx - \int_o^\pi x \sin x\,dx$ [2 marks]

(ii) $\int_o^\pi x \sin x\,dx = \pi$. [5 marks]

Total 9 marks

END OF TEST