FORM TP 2007247

TEST CODE **02134020**

MAY/JUNE 2007

CARIBBEAN EXAMINATIONS COUNCIL

ADVANCED PROFICIENCY EXAMINATION

PURE MATHEMATICS

UNIT 1 – PAPER 02

2 hours

23 MAY 2007 (p.m.)

This examination paper consists of THREE sections: Module 1, Module 2 and Module 3.

Each section consists of 2 questions.
The maximum mark for each section is 40.
The maximum mark for this examination is 120.
This examination consists of 6 pages.

INSTRUCTIONS TO CANDIDATES

1. **DO NOT** open this examination paper until instructed to do so.
2. Answer **ALL** questions from the **THREE** sections.
3. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to three significant figures.

Examination Materials

Mathematical formulae and tables
Electronic calculator
Graph paper

Section A (Module 1)

Answer BOTH questions.

1. (a) Let $g(x) = x^4 - 9, x \in \mathbf{R}$. Find

   (i) **all** the **real** factors of $g(x)$ **[3 marks]**

   (ii) **all** the **real** roots of $g(x) = 0$. **[1 mark]**

   (b) The function f is defined by $f(x) = x^4 - 9x^3 + 28x^2 - 36x + 16, x \in \mathbf{R}$; and $u = x + \frac{4}{x}, x \neq 0$.

   (i) Express u^2 in terms of x. **[3 marks]**

   (ii) By writing $f(x) = x^2\left[x^2 - 9x + 28 - \frac{36}{x} + \frac{16}{x^2}\right]$ and using the result from (b) (i) above, show that if $f(x) = 0$, then $u^2 - 9u + 20 = 0$. **[6 marks]**

   (iii) Hence, determine the values of $x \in \mathbf{R}$ for which $f(x) = 0$. **[7 marks]**

Total 20 marks

2. (a) Let $S_n = \sum_{r=1}^{n} r$ for $n \in \mathbf{N}$. Find the value of n for which $3S_{2n} = 11\,S_n$. **[4 marks]**

   $\left[\textbf{Note:} \sum_{r=1}^{n} r = \frac{1}{2}n(n+1)\right]$

   (b) The quadratic equation $x^2 - px + 24 = 0, p \in \mathbf{R}$, has roots α and β, and the quadratic equation $x^2 - 8x + q = 0, q \in \mathbf{R}$, has roots $2\alpha + \beta$ and $2\alpha - \beta$.

   (i) Express p and q in terms of α and β. **[2 marks]**

   (ii) Find the values of α and β. **[4 marks]**

   (iii) Hence, determine the values of p and q. **[2 marks]**

   (c) Prove, by Mathematical Induction, that $n^2 > 2n$ for all integers $n \geq 3$. **[8 marks]**

Total 20 marks

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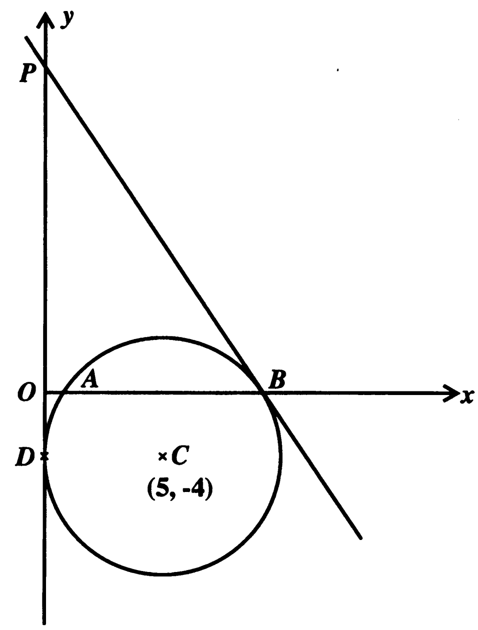
Section B (Module 2)

Answer BOTH questions.

3. The circle shown in the diagram below **(not drawn to scale)** has centre C at $(5, -4)$ and touches the y-axis at the point D. The circle cuts the x-axis at points A and B. The tangent at B cuts the y-axis at the point P.



(a) Determine

(i) the length of the radius of the circle [2 marks]

(ii) the equation of the circle [1 mark]

(iii) the coordinates of the points A and B, at which the circle cuts the x-axis [6 marks]

(iv) the equation of the tangent at B [4 marks]

(v) the coordinates of P. [2 marks]

(b) Show by calculation that $PD = PB$. [5 marks]

Total 20 marks

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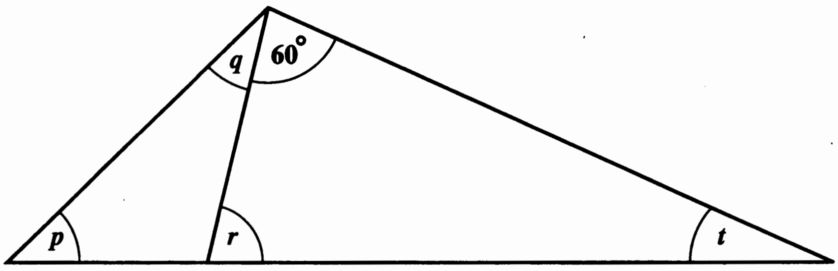
4. (a) (i) Prove that $\cos 2\theta \equiv \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$. **[4 marks]**

(ii) Hence, show, without using calculators, that $\tan 67\frac{1}{2}^\circ = 1+\sqrt{2}$. **[7 marks]**

(b) In the triangle shown below, **(not drawn to scale)**, $\sin q = \frac{3}{5}$ and $\cos p = \frac{5}{13}$.



Determine the **exact** values of

(i) $\cos q$ **[1 mark]**

(ii) $\sin p$ **[1 mark]**

(iii) $\sin r$ **[3 marks]**

(iv) $\cos (p + t)$. **[4 marks]**

Total 20 marks

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Section C (Module 3)

Answer BOTH questions.

5. (a) Given that $y = \sqrt{5x^2 + 3}$,

(i) obtain $\frac{dy}{dx}$ **[4 marks]**

(ii) show that $y\frac{dy}{dx} = 5x$ **[2 marks]**

(iii) hence, or otherwise, show that $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 5$. **[4 marks]**

(b) At a certain port, high tides and low tides occur daily. Suppose t minutes after high tide, the height, h metres, of the tide above a fixed point is given by

$$h = 2\left(1 + \cos\frac{\pi t}{450}\right),\ 0 \leq t.$$

[Note: High tide occurs when h has its maximum value and low tide when h has its minimum value.]

Determine

(i) the height of the tide when high tide occurs for the first time **[2 marks]**

(ii) the length of time which elapses between the first high tide and the first low tide **[3 marks]**

(iii) the rate, in metres per minute, at which the tide is falling 75 minutes after high tide. **[5 marks]**

Total 20 marks

6. (a) (i) Use the result $\int_{o}^{a} f(x)dx = \int_{o}^{a} f(a-x)\,dx, a > 0$, to show that if $I = \int_{0}^{\pi/2} \sin^2 x\,dx$, then $I = \int_{0}^{\pi/2} \cos^2 x\,dx$. **[2 marks]**

(ii) Hence, or otherwise, show that $I = \frac{\pi}{4}$. **[6 marks]**

(b) (i) Sketch the curve $y = x^2 + 4$. **[4 marks]**

(ii) Calculate the volume created by rotating the plane figure bounded by $x = 0$, $y = 4$, $y = 5$ and the curve $y = x^2 + 4$ through 360° **about the y-axis.** **[8 marks]**

Total 20 marks

END OF TEST