Caribbean Examinations Council

Advanced Proficiency Examination

Pure Mathematics Unit 1

Specimen Paper 01

$1\,{}^{1}/_{2}$ hours

Read The Following Instructions Carefully

1. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.

2. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

The lines $2y - 3x - 13 = 0$ and $y + x + 1 = 0$ intersect at the point

(A) (-3, -2) (B) (3, 2) (C) (3, -2) (D) (-3, 2)

Sample Answer

(A) (B) (C) ■

The best answer to this item is " (-3, 2) ", so answer (D) has been blackened.

3. If you want to change your answer, be sure to erase your old answer completely and fill in your new choice.

4. When you are told to begin, turn the page and work as quickly and as carefully as you can. If you cannot answer an item, omit it and go on to the next one. You can come back to the harder item later.

5. You may do any rough work in this booklet.

6. The use of non-programmable calculators is allowed.

7. This test consists of 45 items. Each correct answer carries one mark.

Module 1

1. Let **p**, **q** and **r** be the propositions
p: Applications should include a driving licence,
q: Applications should include a passport-sized photograph,
r: Applications should include an identity card.
The compound proposition, Applications should include a driving licence or identity card (but not both) together with a passport-sized photograph is expressed as

(A) $((\mathbf{p} \wedge \mathbf{r}) \vee \sim (\mathbf{p} \wedge \mathbf{r})) \wedge \mathbf{q}$

(B) $((\mathbf{p} \vee \mathbf{r}) \vee \sim (\mathbf{p} \wedge \mathbf{r})) \vee \mathbf{q}$

(C) $((\mathbf{p} \vee \mathbf{r}) \vee \sim (\mathbf{p} \vee \mathbf{r})) \wedge \mathbf{q}$

(D) $((\mathbf{p} \vee \mathbf{r}) \wedge \sim (\mathbf{p} \wedge \mathbf{r})) \wedge \mathbf{q}$

2. The compound proposition $\mathbf{p} \wedge \mathbf{q}$ is true can be illustrated by the truth table

(A)

p	**q**	$\mathbf{p} \wedge \mathbf{q}$
0	0	0
0	1	0
1	0	0
1	1	1

(B)

p	**q**	$\mathbf{p} \wedge \mathbf{q}$
0	0	1
0	1	0
1	0	0
1	1	1

(C)

p	**q**	$\mathbf{p} \wedge \mathbf{q}$
0	0	0
0	1	0
1	0	1
1	1	1

(D)

p	**q**	$\mathbf{p} \wedge \mathbf{q}$
0	0	1
0	1	0
1	0	1
1	1	0

3. The contrapositive for the conditional proposition $\mathbf{p} \rightarrow \mathbf{q}$ is

(A) $\mathbf{q} \rightarrow \mathbf{p}$

(B) $\sim \mathbf{p} \rightarrow \mathbf{q}$

(C) $\sim \mathbf{q} \rightarrow \sim \mathbf{p}$

(D) $\mathbf{p} \rightarrow \sim \mathbf{q}$

4. The proposition **p**: Sales are decreasing, **q**: Prices are rising, are logically equivalent if

(A) $\sim \mathbf{p} \wedge \sim \mathbf{q}$ (B) $\sim \mathbf{p} \vee \sim \mathbf{q}$

(C) $\sim \mathbf{q} \wedge \mathbf{p}$ (D) $\mathbf{q} \wedge \sim \mathbf{p}$

5. The expression $\dfrac{5\sqrt{45} - \sqrt{80}}{\sqrt{5} - \sqrt{125}}$ is equal to

(A) $\dfrac{5\sqrt{35}}{\sqrt{120}}$ (B) $\dfrac{11\sqrt{5}}{5}$

(C) $\dfrac{11}{4\sqrt{5}}$ (D) $-\dfrac{11}{4}$

6. If $2x^3 + ax^2 - 5x - 1$ leaves a remainder of 3 when divided by $(2x + 1)$, then a is

(A) -7 (B) 7

(C) $-\frac{1}{2}$ (D) -5

7. Given that $x = 3^y$, $y > 0$ then $\log_x 3$ is equal to

(A) y (B) $3y$

(C) $\dfrac{1}{y}$ (D) $\dfrac{3}{y}$

8. For $x < 2$ the inverse function, $f^{-1}(x)$, of $f(x) = 2 - e^{2x}$ is

(A) $\ln(2 - x)$ (B) $\ln(2 - 2x)$

(C) $2\ln(2 - x)$ (D) $\frac{1}{2}\ln(2 - x)$

9. The function f $(x) = 2x^2 - 4x + 5$, for $x \in \mathbb{R}$, is one-to-one for $x > k$, where $k \in \mathbb{R}$. The value of k is

(A) 3 (B) 1

(C) -1 (D) -2

10. Given that fg $(x) = x$, where g $(x) = \frac{2x+1}{3}$, f $(x) =$

(A) $\frac{3x-1}{2}$ (B) $\frac{3}{2x+1}$

(C) $\frac{2}{3x+1}$ (D) $\frac{3x}{2x+1}$

11. The values of x for which $|2x - 3| = x + 1$ are

(A) $x = \frac{2}{3}$, $x = 4$ (B) $x = \frac{2}{3}$, $x = -4$

(C) $x = -\frac{2}{3}$, $x = -4$ (D) $x = -\frac{2}{3}$, $x = 4$

12. Given that $x^2 + 4x + 3$ is a factor of f $(x) = 2x^3 + 7x^2 + 2x - 3$, the values of x for which f $(x) = 0$ are

(A) $-3, 1, -\frac{1}{2}$ (B) $-3, -1, -\frac{1}{2}$

(C) $3, 1, \frac{1}{2}$ (D) $-3, -1, \frac{1}{2}$

13. The cubic equation $2x^3 + x^2 - 22x + 24 = 0$ has roots α, β and γ. The value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

(A) $-\frac{1}{11}$ (B) $-\frac{1}{12}$

(C) $\frac{11}{12}$ (D) $-\frac{1}{2}$

14. The range(s) of values of x for which $\frac{3x+2}{x-1} > 0$ are

(A) $x > -\frac{2}{3}$, $x > 1$ (B) $-\frac{2}{3} < x < 1$

(C) $x < \frac{2}{3}$, $x > 1$ (D) $x < -\frac{2}{3}$, $x > 1$

15. The values of x for which $|x + 5| > 3$ are

(A) $x < -8, x < -2$ (B) $x > 0, x < 1$

(C) $x > -2$, $x < -8$ (D) $x > -2, x > -8$

16. $\frac{1+\cot^2\theta}{\sec\theta\operatorname{cosec}\theta} \equiv$

(A) $\tan\theta$ (B) $\operatorname{cosec}\theta$

(C) $\cot\theta$ (D) $\cos\theta$

17. The general solution of the equation $\cos 2\theta = 1$ is

(A) $n\pi + \frac{\pi}{4}$ (B) $n\pi$

(C) $n\pi + \frac{\pi}{2}$ (D) $\frac{(2n+1)\pi}{4}$

18. If $\cos A = \frac{3}{5}$ and A is acute, then $\sin 2A$ is equal to

(A) $\frac{8}{25}$ (B) $\frac{24}{25}$

(C) $\frac{6}{25}$ (D) $\frac{12}{25}$

19. $\cos\theta + 3\sin\theta = 2$ can be expressed as

(A) $4\cos\left(\theta - \tan^{-1}\left(\frac{1}{3}\right)\right) = 2$

(B) $2\cos\left(\theta + \tan^{-1}(3)\right) = 2$

(C) $\sqrt{10}\cos\left(\theta - \tan^{-1}(3)\right) = 2$

(D) $\sqrt{10}\cos\left(\theta + \tan^{-1}\left(\frac{1}{3}\right)\right) = 2$

20. The minimum value of $\dfrac{1}{2\cos\left(\theta + \frac{\pi}{4}\right)}$ is

(A) $\frac{1}{2}$

(B) 2

(C) -1

(D) 0

21. A curve C_1 is given by the equation $y = x^2 + 1$, and a curve C_2 is given by the equation $\frac{16}{x^2} + 1$, $x \in \mathbb{R}$, $x > 0$. The value of x for which $C_1 = C_2$ is

(A) -2

(B) 4

(C) -4

(D) 2

22 The tangent to the circle, C, with equation $x^2 + y^2 + 4x - 10y - 5 = 0$ at the point $P(3, 2)$ has equation

(A) $3x + 5y - 19 = 0$

(B) $5x + 3y + 19 = 0$

(C) $3x - 5y + 19 = 0$

(D) $5x - 3y - 9 = 0$

23. The x-coordinates of the points where the line l with equation $y = 2x + 1$ cuts the curve C with equation $\frac{6}{x}$ are

(A) $\frac{3}{2}, -2$

(B) $-\frac{3}{2}, -2$

(C) $\frac{3}{2},\ 2$

(D) $-\frac{3}{2},\ 2$

24. The Cartesian equation of the curve C given by the parametric equations $x = 3\sin\theta - 2$, $y = 4\cos\theta + 3$ is

(A) $9(x-3)^2 + 4(y-4)^2 = 36$ (B) $x^2 + y^2 = 30$

(C) $9x^2 + 16y^2 = 13$ (D) $16(x+2)^2 + 9(y-3)^2 = 144$

25. Relative to a fixed origin O the position vector of A is $\overrightarrow{\mathrm{OA}} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the position vector of B is $\overrightarrow{\mathrm{OB}} = 9\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$. $|\overrightarrow{\mathrm{AB}}|$ is

(A) 49 units (B) 1 unit

(C) $3\sqrt{21}$ units (D) 7 units

26. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, and point B has position vector $(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$.The line l passes through the points A and B. A vector equation for the line l is given by

(A) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-7\mathbf{i} + 6\mathbf{j} - \mathbf{k})$

(B) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-5\mathbf{i} + 9\mathbf{j} - 5\mathbf{k})$

(C) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-3\mathbf{i} + 12\mathbf{j} - 9\mathbf{k})$

(D) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \lambda(-10\mathbf{i} + 27\mathbf{j} + 20\mathbf{k})$

27. Relative to a fixed origin O, the line l_1 has position vector $\begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, and the line l_2 has position vector $\begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \mu\begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$, where λ and μ are scalars.

The cosine of the acute angle between l_1 and l_2 is given by

(A) $\cos\theta = -\frac{2}{3}$ (B) $\cos\theta = \frac{2}{3}$

(C) $\cos\theta = \frac{8}{27}$ (D) $\cos\theta = \frac{2}{27}$

28. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point B has position vector $(5\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$. Given that a vector $\mathbf{v}$ is of magnitude $3\sqrt{6}$ units in the direction of $|\overrightarrow{AB}|$, then $\mathbf{v}$ =

(A) 3i + 3j – 6k (B) −3i – 3j + 6k

(B) −3i + 3j + 6k (D) 3i – 3j + 6k

29. The point P (3, 0, 1) lies in the plane Π with equation $\mathbf{r} \cdot (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) = d$. The constant d is

(A) $\sqrt{21}$ (B) 10

(C) 5 (D) $\sqrt{41}$

30. The line l_1 has equation $\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$ and the line l_2 has equation $-5\mathbf{i} + 15\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + a\mathbf{k})$. Given that l_1 is perpendicular to l_2 the value of a is

(A) 2 (B) –2

(C) –4 (D) 6

31. $\lim\limits_{x \to 3} \dfrac{2x^2 - 5x - 3}{x^2 - 2x - 3}$ is

(A) $\dfrac{7}{4}$ (B) $\propto$

(C) 0 (D 1

32. $\lim\limits_{\theta \to 0} \dfrac{\sin x}{\frac{x}{2}}$ is

(A) 1 (B) 0

(C) $\dfrac{1}{2}$ (D) 2

33. Given that $\lim_{x \to -1} \{3f(x) + 2\} = 11$, where f (x) is real and continuous, the $\lim_{x \to -1} \{2f(x) + 5x\}$ is

(A) 4 (B) 13

(C) 1 (D) −11

34. Given that f (x) = $(3x + 2)(2 \sin x)$, then $f'(x)$ is

(A) $6 \cos x$ (B) $2(3x + 2) \cos x + 6 \sin x$

(C) $6x \cos x + 6 \sin x$ (D) $3 + 2 \sin x + (3x + 2) \cos x$

35. The derivative by first principles of the function f $(x) = \frac{1}{x^2}$ is given by

(A) $\lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$ (B) $\lim_{h \to 0} \frac{\frac{1}{(x^2+h)} - \frac{1}{x^2}}{h}$

(C) $\lim_{h \to 0} \frac{\frac{1}{x^2} - \frac{1}{(x^2+h)}}{h}$ (D) $\lim_{h \to 0} \frac{1}{(x+h)^2} - \frac{1}{x^2}$

36. Given f (x) = 3 cos 2x, then $f^{-1}(x)$ =

(A) $6 \sin 2x$ (B) $-3 \sin 2x$

(C) $-6 \sin 2x$ (D) $-\frac{3}{2} \sin 2x$

37. The curve C with equation $y = x^3 - 6x^2 + 9x$ has stationary points at P (3, 0) and Q $(1, 4)$. The nature of these stationary points are

(A) $(3, 0)_{\max}$ $(1, 4)_{\min}$ (B) $(3, 0)_{\min}$ $(1, 4)_{\max}$

(C) $(3, 0)_{\text{infl}}$ $(1, 4)_{\max}$ (D) $(3, 0)_{\text{infl}}$ $(1, 4)_{\min}$

38. Given that the gradient function to a curve C at the point P (2, 3) is $6x^2 - 14x$, the equation of the normal to C at P is given by the equation

(A) $y - 3 = -2(x - 2)$

(B) $y - 3 = -4(x - 2)$

(C) $y - 3 = 4(x - 2)$

(D) $y - 3 = \frac{1}{4}(x - 2)$

39. $\int \frac{(2x + 1)^2}{\sqrt{x}} \, dx =$

(A) $\int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$

(B) $\int \left(4x^{\frac{3}{2}} + x^{-\frac{1}{2}} \right) dx$

(C) $\int \left(4x + 4 + x^{-1} \right) dx$

(D) $\int \left(4x^{\frac{5}{2}} + 4x^{\frac{3}{2}} + x^{\frac{1}{2}} \right) dx$

40. Given that $\int_1^3 f(x)dx = 8$, then $\int_1^3 [2f(x) - 5]dx =$

(A) 11

(B) 21

(C) 13

(D) 6

41. The area of the finite region, R, enclosed by the curve $y = x - \frac{1}{\sqrt{x}}$, the lines $x = 1$ and $x = 4$ is

(A) $\frac{5}{2}$

(B) $\frac{11}{2}$

(C) $\frac{27}{4}$

(D) $\frac{19}{2}$

42. The region, R, enclosed by the curve with equation $y = 4 - x^2$ in the first quadrant is rotated completely about the ***y*-axis**. The volume of the solid generated is given by

(A) $\pi \int_0^4 (4 - y)\, \mathrm{d}y$

(B) $\pi \int_0^4 (4 - y)^2\, \mathrm{d}y$

(C) $\pi \int_0^2 (4 - y)\, \mathrm{d}y$

(D) $\pi \int_0^2 (4 - y)^2\, \mathrm{d}y$

43. Given that $\dfrac{\mathrm{d}}{\mathrm{d}x}\, \dfrac{2x-1}{3x+2} = \dfrac{7}{(3x+2)^2}$ then $\displaystyle\int_1^2 \frac{21}{(3x+2)^2}\, \mathrm{d}x =$

(A) $\left.\dfrac{3(7)}{(3x+2)^2}\right|_1^2$

(B) $\left.\dfrac{2x-1}{(3x+2)}\right|_1^2$

(C) $\left.\dfrac{3(2x-1)}{(3x+2)}\right|_1^2$

(D) $\left.\dfrac{-21}{(3x+2)}\right|_1^2$

44. Given that $\displaystyle\int_{-2}^{0} \mathrm{f}(x)\, \mathrm{d}x = \frac{16}{3}$ and $\displaystyle\int_{-2}^{2} \mathrm{f}(x)\, \mathrm{d}x = \frac{32}{3}$ where $\mathrm{f}(x)$ is a real continuous function in the closed interval $[-2, 2]$, then $\displaystyle\int_0^2 \mathrm{f}(x)\, \mathrm{d}x =$

(A) 16

(B) $\dfrac{16}{3}$

(C) $\dfrac{64}{3}$

(D) 32

45. Water is pumped into a large tank at a rate that is proportional to its volume, V, at time t seconds. There is a small hole at the bottom of the tank and water leaks out at a constant rate of 5 m^3/s. Given that k is a positive constant, a differential equation that satisfies this situation is

(A) $\frac{dV}{dt} = kV - 5$ (B) $\frac{dV}{dt} = -kV - 5$

(C) $\frac{dV}{dt} = kV + 5$ (D) $\frac{dV}{dt} = kV$

End of Test

Key

Unit 1 Paper 01

Module	Item	Key	S. O.	Module	Item	Key	S. O.
1	1	D	A1	3	31	A	A5
	2	A	A2		32	D	A6
	3	C	A3		33	C	A4
	4	B	A4		34	B	B3
	5	D	B3		35	A	B2
	6	B	C1		36	C	B5
	7	C	D6		37	B	B9
	8	D	D4		38	D	B16
	9	B	E1		39	A	C2
	10	A	E4		40	D	C4 (ii)
	11	A	F3		41	B	C8 (i)
	12	D	G1		42	A	C8 (iii)
	13	C	G2		43	C	C5 (i)
	14	D	H1		44	B	C7 (iii)
	15	C	H2		45	A	C9 (i)
2	16	C	A5				
	17	B	A8				
	18	B	A3				
	19	C	A7				
	20	A	A10				
	21	D	B3				
	22	D	B1				
	23	A	B2				
	24	D	B4				
	25	D	C6				
	26	A	C8				
	27	B	C7				
	28	B	C5				
	29	C	C10				
	30	A	C9				