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PURE MATHEMATICS

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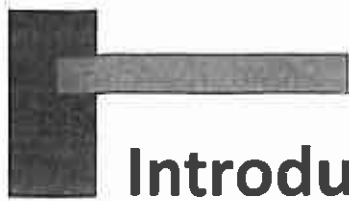
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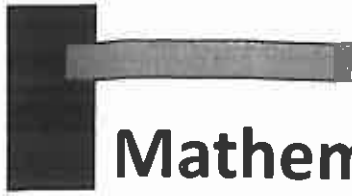


Introduction

The Caribbean Advanced Proficiency Examination (CAPE) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and knowledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under CAPE may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification. The first is the award of a certificate showing each CAPE Unit completed. The second is the CAPE Diploma, awarded to candidates who have satisfactorily completed at least six Units including Caribbean Studies. The third is the CAPE Associate Degree, awarded for the satisfactory completion of a prescribed cluster of seven CAPE Units including Caribbean Studies and Communication Studies. For the CAPE Diploma and the CAPE Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years.

Recognised educational institutions presenting candidates for CAPE Associate Degree in one of the nine categories must, on registering these candidates at the start of the qualifying year, have them confirm in the required form, the Associate Degree they wish to be awarded. Candidates will not be awarded any possible alternatives for which they did not apply.



Mathematics Syllabus

◆ RATIONALE

Mathematics is one of the oldest and most universal means of creating, communicating, connecting and applying structural and quantitative ideas. The discipline of Mathematics allows the formulation and solution of real-world problems as well as the creation of new mathematical ideas, both as an intellectual end in itself, but also as a means to increase the success and generality of mathematical applications. This success can be measured by the quantum leap that occurs in the progress made in other traditional disciplines once mathematics is introduced to describe and analyse the problems studied. It is therefore essential that as many persons as possible be taught not only to be able to use mathematics, but also to understand it.

Students doing this syllabus will have been already exposed to Mathematics in some form mainly through courses that emphasise skills in using mathematics as a tool, rather than giving insight into the underlying concepts. To enable students to gain access to mathematics training at the tertiary level, to equip them with the ability to expand their mathematical knowledge and to make proper use of it, it is necessary that a mathematics course at this level should not only provide them with more advanced mathematical ideas, skills and techniques, but encourage them to understand the concepts involved, why and how they "work" and how they are interconnected. It is also to be hoped that, in this way, students will lose the fear associated with having to learn a multiplicity of seemingly unconnected facts, procedures and formulae. In addition, the course should show them that mathematical concepts lend themselves to generalisations, and that there is enormous scope for applications to the solving of real problems.

Mathematics covers extremely wide areas. However, students can gain more from a study of carefully selected, representative areas of Mathematics, for a "mathematical" understanding of these areas, rather than to provide them with only a superficial overview of a much wider field. While proper exposure to a mathematical topic does not immediately make students into experts in it, that proper exposure will certainly give the students the kind of attitude which will allow them to become experts in other mathematical areas to which they have not been previously exposed. The course is, therefore, intended to provide quality in selected areas rather than in a large number of topics.

To optimise the competing claims of spread of syllabus and the depth of treatment intended, all items in the proposed syllabus are required to achieve the aims of the Course. While both Units 1 and 2 can stand on their own, it is advisable that students should complete Unit 1 before doing Unit 2, since Unit 1 contains basic knowledge for students of tertiary level courses including Applied Mathematics.

Through a development of understanding of these areas, it is expected that the course will enable students to:

1. develop mathematical thinking, understanding and creativity;
2. develop skills in using mathematics as a tool for other disciplines;
3. develop the ability to communicate through the use of mathematics;
4. develop the ability to use mathematics to model and solve real-world problems;
5. gain access to mathematics programmes at tertiary institutions.

◆ AIMS

The syllabus aims to:

1. provide understanding of mathematical concepts and structures, their development and the relationships between them;
2. enable the development of skills in the use of mathematical tools;
3. develop an appreciation of the idea of mathematical proof, the internal logical coherence of Mathematics, and its consequent universal applicability;
4. develop the ability to make connections between distinct concepts in Mathematics, and between mathematical ideas and those pertaining to other disciplines;
5. develop a spirit of mathematical curiosity and creativity, as well as a sense of enjoyment;
6. enable the analysis, abstraction and generalisation of mathematical ideas;
7. develop in students the skills of recognising essential aspects of concrete real-world problems, formulating these problems into relevant and solvable mathematical problems and mathematical modelling;
8. develop the ability of students to carry out independent or group work on tasks involving mathematical modelling;
9. provide students with access to more advanced courses in Mathematics and its applications at tertiary institutions.

◆ SKILLS AND ABILITIES TO BE ASSESSED

The assessment will test candidates' skills and abilities in relation to three cognitive levels.

1. **Conceptual knowledge** is the ability to **recall, select and use** appropriate facts, concepts and principles in a variety of contexts.
2. **Algorithmic knowledge** is the ability to **manipulate** mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.
3. **Reasoning** is the ability to **select** appropriate strategy or **select, use and evaluate** mathematical models and **interpret** the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.

◆ PRE-REQUISITES OF THE SYLLABUS

Any person with a **good** grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Mathematics, *and/or the Caribbean Secondary Education Certificate (CSEC) General Proficiency course in Additional Mathematics*, or equivalent, should be able to undertake the course. However, successful participation in the course will also depend on the possession of good verbal and written communication skills.

◆ STRUCTURE OF THE SYLLABUS

The syllabus is arranged into two (2) Units, Unit 1 which will lay foundations, and Unit 2 which expands on, and applies, the concepts formulated in Unit 1.

It is, therefore, recommended that Unit 2 be taken after satisfactory completion of Unit 1 or a similar course. Completion of each Unit will be separately certified.

Each Unit consists of three Modules.

Unit 1: Algebra, Geometry and Calculus, contains three Modules, each requiring approximately 50 hours. The total teaching time, therefore, is approximately 150 hours.

- Module 1 - Basic Algebra and Functions
- Module 2 - *Trigonometry, Geometry and Vectors*
- Module 3 - Calculus I

Unit 2: *Complex Numbers, Analysis and Matrices* contains three Modules, each requiring approximately 50 hours. The total teaching time, therefore, is approximately 150 hours.

- Module 1 - *Complex Numbers and Calculus II*
- Module 2 - Sequences, Series and Approximations
- Module 3 - *Counting, Matrices and Differential Equations*

◆ RECOMMENDED 2-UNIT OPTIONS

1. Pure Mathematics Unit 1 **AND** Pure Mathematics Unit 2.
2. Applied Mathematics Unit 1 **AND** Applied Mathematics Unit 2.
3. Pure Mathematics Unit 1 **AND** Applied Mathematics Unit 2.

◆ MATHEMATICAL MODELLING

Mathematical Modelling should be used in both Units 1 and 2 to solve real-world problems.

A. The topic Mathematical Modelling involves the following steps:

1. identification of a real-world situation to which modelling is applicable;
2. carry out the modelling process for a chosen situation to which modelling is applicable;
3. discuss and evaluate the findings of a mathematical model in a written report.

B. The Modelling process requires:

1. a clear statement posed in a real-world situation, and identification of its essential features;
2. translation or abstraction of the problem, giving a representation of the essential features of the real-world;
3. solution of the mathematical problem (analytic, numerical, approximate);
4. testing the appropriateness and the accuracy of the solution against behaviour in the real-world;
5. refinement of the model as necessary.

C. Consider the two situations given below.

1. A weather forecaster needs to be able to calculate the possible effects of atmospheric pressure changes on temperature.
2. An economic adviser to the Central Bank Governor needs to be able to calculate the likely effect on the employment rate of altering the Central Bank's interest rate.

In each case, people are expected to predict something that is likely to happen in the future. Furthermore, in each instance, these persons may save lives, time, and money or change their actions in some way as a result of their predictions.

One method of predicting is to set up a mathematical model of the situation. A mathematical model is not usually a model in the sense of a scale model motor car. A mathematical model is a way of describing an underlying situation mathematically, perhaps with equations, with rules or with diagrams.

D. Some examples of mathematical models are:

1. Equations

(a) Business

A recording studio invests \$25 000 to produce a master CD of a singing group. It costs \$50.00 to make each copy from the master and cover the operating expenses. We can model this situation by the equation or mathematical model,

$$C = 50.00x + 25\,000$$

where C is the cost of producing x CDs. With this model, one can predict the cost of producing 60 CDs or 6 000 CDs.

Is the equation $x + 2 = 5$ a mathematical model? Justify your answer.

(b) Banking

Suppose you invest \$100.00 with a commercial bank which pays interest at 12% per annum. You may leave the interest in the account to accumulate. The equation $A = 100(1.12)^n$ can be used to model the amount of money in your account after n years.

2. Table of Values

Traffic Management

The table below shows the safe stopping distances for cars recommended by the Highway Code.

Speed <i>m/h</i>	Thinking Distance m	Braking Distance m	Overall Stopping Distance m
20	6	6	12
30	9	14	23
40	12	24	36
50	15	38	53
60	18	55	73
70	21	75	96

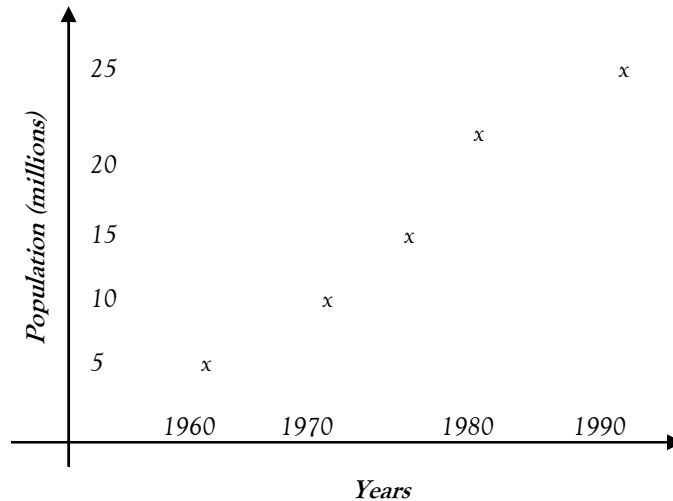
We can predict our stopping distance when travelling at 50 m/h from this model.

3. Rules of Thumb

You might have used some mathematical models of your own without realizing it; perhaps you think of them as “rules of thumb”. For example, in the baking of hams, most cooks used the rule of thumb that “bake ham fat side up in roasting pan in a moderate oven (160°C) ensuring 25 to 40 minutes per ½kg”. The cook is able to predict how long it takes to bake his ham without burning it.

4. Graphs

Not all models are symbolic in nature; they may be graphical. For example, the graph below shows the population at different years for a certain country.



RESOURCE

Hartzler, J. S. and Swetz, F.

Mathematical Modelling in the Secondary School Curriculum, A Resource Guide of Classroom Exercises, Vancouver, United States of America: National Council of Teachers of Mathematics, Incorporated, Reston, 1991.

◆ UNIT 1 - ALGEBRA, GEOMETRY AND CALCULUS

MODULE 1: BASIC ALGEBRA AND FUNCTIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. develop the ability to construct simple proofs of mathematical assertions;
2. understand the concept of a function;
3. be confident in the manipulation of algebraic expressions and the solutions of equations and inequalities;
4. *understand the properties and significance of the exponential and logarithm function;*
5. develop the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) *Reasoning and Logic*

Students should be able to:

1. *establish the truth value of compound statements using truth tables;*
2. *state the converse, contrapositive and inverse of a conditional (implication) statement;*
3. *determine whether two statements are logically equivalent.*

CONTENT

(A) *Reasoning and Logic*

1. *Simple statement (proposition), connectives (conjunction, disjunction, negation, conditional, bi-conditional), compound statements.*
2. *Truth tables.*
3. *Converse and contrapositive of statements.*
4. *Logical equivalence.*

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

SPECIFIC OBJECTIVES

(B) The Real Number System – \mathbb{R}

Students should be able to:

1. *perform binary operations;*
2. *use the concepts of identity, closure, inverse, commutativity, associativity, distributivity addition, multiplication and other simple binary operations;*
3. *perform operations involving surds;*
4. construct simple proofs, specifically direct proofs, or proof by the use of counter examples;
5. establish simple proofs by using the principle of mathematical induction.

CONTENT

(B) The Real Number System – \mathbb{R}

1. *Definition of binary operation.*
2. *Definition of commutativity, associativity, distributivity, identity, inverse and closure.*
3. Axioms of the system - including commutative, associative and distributive laws; non-existence of the multiplicative inverse of zero.
4. Methods of proof - direct, counter-examples.
5. Simple applications of mathematical induction.

SPECIFIC OBJECTIVES

(C) *Algebraic Operations*

Students should be able to:

1. *apply the Remainder Theorem;*
2. use the Factor Theorem to find factors and to evaluate unknown coefficients;
3. extract all factors of $a^n - b^n$ for positive integers $n \leq 6$;

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

4. use the concept of identity of polynomial expressions;
5. *sketch graphs of simple polynomial and rational expressions;*
6. *solve graphically equations involving polynomial and rational expressions.*

CONTENT

(C) Algebraic Operations

1. Factor Theorem.
2. Remainder Theorem.
3. *Graphs of simple polynomials and rational expressions.*
4. *Solution of rational equations.*
5. *Curve sketching.*

SPECIFIC OBJECTIVES

(D) Exponential and Logarithmic Functions

Students should be able to:

1. *define an exponential function $y = a^x$ for $a \in \mathbb{R}$;*
2. *sketch the graph of $y = a^x$;*
3. *define a logarithmic function as the inverse of an exponential function;*
4. *define the exponential functions $y = e^x$ and its inverse $y = \ln x$, where $\ln x \equiv \log_e x$;*
5. *use the fact that $y = \ln x \Leftrightarrow x = e^y$;*
6. *simplify expressions by using laws of logarithms;*
7. *use logarithms to solve equations of the form $a^x = b$;*
8. solve problems involving changing of the base of a logarithm.

UNIT 1
MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

CONTENT

(D) Exponential and Logarithmic Functions

1. Graphs of the functions a^x and $\log_a x$.
2. Properties of the exponential and logarithmic functions.
3. Exponential and natural logarithmic functions and their graphs.
4. Laws of logarithms applied to problems:
 - (a) $\ln(PQ) = \ln P + \ln Q$;
 - (b) $\ln(P/Q) = \ln P - \ln Q$;
 - (c) $\ln P^a = a \ln P$.

SPECIFIC OBJECTIVES

(E) Functions

Students should be able to:

1. *define mathematically the terms: function, domain, range, one-to-one function (injective function), onto function (surjective function), many-to-one, one-to-one and onto function (bijective function), composition and inverse of functions;*
2. *prove whether or not a given simple function is one-to-one or onto and if its inverse exists;*
3. *use the fact that a function may be defined as a set of ordered pairs;*
4. *use the fact that if g is the inverse function of f , then $f[g(x)] = x$, for all x , in the domain of g ;*
5. show graphical solutions of $f(x) = g(x)$, $f(x) \leq g(x)$, $f(x) \geq g(x)$;
6. identify an increasing or decreasing function, using the sign of $\frac{f(a) - f(b)}{a - b}$,
when $a \neq b$;
7. illustrate by means of graphs, the relationship between the function $y = f(x)$ given in graphical form and $y = af(x)$; $y = f(x \pm a)$; $y = f(x) \pm a$; $y = af(x \pm b)$;
 $y = f(ax)$, $y = |f(x)|$, where a, b are real numbers, and, where it is invertible,
 $y = f^{-1}(x)$.

UNIT 1
MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

CONTENT

(E) Functions

1. Domain, range, *many-to-one*, composition.
2. Injective, surjective, bijective functions, inverse function.
3. Graphical solutions of problems involving functions.
4. Transformation of the graph $y = f(x)$ to $y = af(x)$; $y = f(x \pm a)$; $y = f(x) \pm a$; $y = af(x \pm b)$; $y = f(ax)$; $y = |f(x)|$ and, if appropriate, to $y = f^{-1}(x)$.

SPECIFIC OBJECTIVES

(F) *The Modulus Function*

Students should be able to:

1. *define the modulus function;*
2. *use the properties:*
 - (a) $|x|$ is the positive square root of x^2 ;
 - (b) $|x| < |y|$ if, and only if, $x^2 < y^2$;
 - (c) $|x| < |y| \Leftrightarrow \text{iff } -y < x < y$;
 - (d) $|x + y| \leq |x| + |y|$.
3. *solve equations involving the modulus functions.*

CONTENT

(F) *The Modulus Function*

1. Definition of the modulus function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases};$$

2. The triangle inequality.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

CONTENT

(F) *The Modulus Function*

3. Definition of the modulus function.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases};$$

4. The triangle inequality.

SPECIFIC OBJECTIVES

(G) *Cubic Functions and Equations*

Students should be able to:

1. find the roots of a cubic equation;
2. use the relationship between the *sum of the roots, the product of the roots, the sum of the product of the roots pair-wise* and the coefficients of $ax^3 + bx^2 + cx + d = 0$.

CONTENT

(G) *Cubic Functions and Equations*

1. *Real Roots of cubic equations.*
2. Sums and products, with applications, of the roots of cubic equations.

SPECIFIC OBJECTIVES

(H) *Inequalities*

Students should be able to:

use algebraic and graphical methods to find solution sets of inequalities of the form

1. $\frac{ax+b}{cx+d} > 0$;

2. $|x+a| < b$;

3. $|ax+b| < |cx+d|$.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

(H) Inequalities

Inequalities involving simple rational and modulus functions.

CONTENT

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. The Real Number System

- (a) *The teacher should do a brief review of the number system before starting this section.*
- (b) *The teacher should encourage students to practice different methods of proof, for example, to prove that the product of two consecutive integers is an even integer.*

2. Proof by Mathematical Induction (MI)

Typical Question

Prove that some formula or statement P is true for all positive integers $n \geq k$, where k is some positive integer; usually $k = 1$.

Procedure

Step 1: Verify that when $k = 1$: P is true for $n = k = 1$. This establishes that P is true for $n = 1$.

Step 2: Assume P is true for $n = k$, where k is a positive integer > 1 . At this point, the statement k replaces n in the statement P and is taken as true.

Step 3: Show that P is true for $n = k + 1$ using the true statement in step 2 with n replaced by k .

Step 4: At the end of step 3, it is stated that statement P is true for all positive integers $n \geq k$.

Summary

Proof by MI: For $k > 1$, verify Step 1 for k and proceed through to Step 4.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

Observation

Most users of MI do not see how this proves that P is true. The reason for this is that there is a massive gap between Steps 3 and 4 which can only be filled by becoming aware that Step 4 only follows because Steps 1 to 3 are repeated an infinity of times to generate the set of all positive integers. The focal point is the few words "for all positive integers $n \geq k$ " which points to the determination of the set S of all positive integers for which P is true.

Step 1 says that $1 \in S$ for $k = 1$.

Step 3 says that $k + 1 \in S$ whenever $k \in S$, so immediately $2 \in S$ since $1 \in S$.

Iterating on Step 3 says that $3 \in S$ since $2 \in S$ and so on, so that $S = \{1, 2, 3 \dots\}$, that is, S is the set of all positive integers when $k = 1$ which brings us to Step 4.

When $k > 1$, the procedure starts at a different positive integer, but the execution of steps is the same. Thus, it is necessary to explain what happens between Steps 3 and 4 to obtain a full appreciation of the method.

Example 1: Use Mathematical Induction to prove that $n^3 - n$ is divisible by 3, whenever n is a positive integer.

Solution: Let $P(n)$ be the proposition that " $n^3 - n$ is divisible by 3".

Basic Step: $P(1)$ is true, since $1^3 - 1 = 0$ which is divisible by 3.

Inductive Step: Assume $P(n)$ is true: that is, $n^3 - n$ is divisible by 3. We must show that $P(n + 1)$ is true, if $P(n)$ is true. That is, is, $(n + 1)^3 - (n + 1)$ is divisible by 3.

$$\begin{aligned} \text{Now, } (n + 1)^3 - (n + 1) &= (n^3 + 3n^2 + 3n + 1) - (n + 1) \\ &= (n^3 - n) + 3(n^2 + n) \end{aligned}$$

Both terms are divisible by 3 since $(n^3 - n)$ is divisible by 3 by the assumption and $3(n^2 + n)$ is a multiple of 3. Hence, $P(n + 1)$ is true whenever $P(n)$ is true.

Thus, $n^3 - n$ is divisible by 3 whenever n is a positive integer.

Example 2: Prove by Mathematical Induction that the sum S_n of the first n odd positive integers is n^2 .

Solution: Let $P(n)$ be the proposition that the sum S_n of the first n odd positive integers is n^2 .

Basic Step: For $n = 1$ the first odd positive integer is 1, so $S_1 = 1$, that is: $S_1 = 1 = 1^2$, hence $P(1)$ is true.

UNIT 1
MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

Inductive Step: Assume $P(n)$ is true. That is, $S_n = 1 + 3 + 5 + \dots + (2n - 1) = n^2$.

$$\begin{aligned} \text{Now, } S_{n+1} &= 1 + 3 + 5 + \dots + (2n - 1) + (2n + 1) \\ &= [1 + 3 + 5 + \dots + (2n - 1)] + (2n + 1) \\ &= n^2 + (2n + 1), \text{ by the assumption,} \\ &= (n + 1)^2 \end{aligned}$$

Thus, $P(n+1)$ is true whenever $P(n)$ is true.

Since $P(1)$ is true and $P(n) \rightarrow P(n + 1)$, the proposition $P(n)$ is true for all positive integers n .

3. Functions (Injective, surjective, bijective) – Inverse Function

Mathematical proof that a function is one-to-one (injective), onto (surjective) or (one-to-one and onto function) bijective should be introduced at this stage.

Teacher and students should explore the mapping properties of quadratic functions which:

- (a) will, or will not, be injective, depending on which subset of the real line is chosen as the domain;
- (b) will be surjective if its range is taken as the co-domain (completion of the square is useful here);
- (c) if both injective and surjective, will have an inverse function which can be constructed by solving a quadratic equation.

Example: Use the function $f : A \rightarrow B$ given by $f(x) = 3x^2 + 6x + 5$, where the domain A is alternatively the whole of the real line, or the set $\{x \in \mathbb{R} \mid x \geq -1\}$, and the co-domain B is \mathbb{R} or the set $\{y \in \mathbb{R} \mid y \geq 2\}$.

4. Cubic Equations

Teachers should first review the theory of the quadratic equation and the nature of its roots.

UNIT 1

MODULE 1: BASIC ALGEBRA AND FUNCTIONS (cont'd)

RESOURCES

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UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop* the ability to represent and deal with objects in two and three dimensions through the use of coordinate geometry and vectors;
2. *develop* the ability to manipulate and describe the behaviour of trigonometric functions;
3. *develop* the ability to establish trigonometric identities;
4. acquire the skills to solve trigonometric equations;
5. *acquire the skills to conceptualise and to manipulate objects in two and three dimensions*;
6. *develop* the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) Trigonometric Functions, Identities and Equations (all angles will be assumed to be in radians unless otherwise stated)

Students should be able to:

1. graph the functions $\sin kx$, $\cos kx$, $\tan kx$, $k \in \mathbb{Q}$;
2. relate the periodicity, symmetries and amplitudes of the functions in Specific Objective 1 above to their graphs;
3. *use compound-angle formulae*;
4. *use the reciprocal functions of $\sec x$, $\operatorname{cosec} x$ and $\cot x$ and their related graphs*;
5. *derive the following identities*:
 - (a) $\sin kA$, $\cos kA$, $\tan kA$, for $k \in \mathbb{Q}$;
 - (b) $\tan^2 x$, $\cot^2 x$, $\sec^2 x$ and $\operatorname{cosec}^2 x$;
 - (c) $\sin A \pm \sin B$, $\cos A \pm \cos B$.
6. *prove further identities using Specific Objective 5*;

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

7. express $a \cos \theta + b \sin \theta$ in the form $r \cos(\theta \pm \alpha)$ and $r \sin(\theta \pm \alpha)$, where r is positive, $0 < \alpha < \frac{\pi}{2}$;
8. find the general solution of equations of the form:
 - (a) $\sin k\theta = s$,
 - (b) $\cos k\theta = c$,
 - (c) $\tan k\theta = t$,
 - (d) $a \cos \theta + b \sin \theta = c$,for $a, b, c, k, \in \mathbb{R}$;
9. find the solutions of the equations in Specific Objectives 8 above for a given range;
10. obtain maximum or minimum values of $f(\theta)$ for $0 \leq \theta \leq 2\pi$.

CONTENT

(A) Trigonometric Functions, Identities and Equations (all angles will be assumed to be radians)

1. The functions $\cot x$, $\sec x$, $\operatorname{cosec} x$.
2. Compound-angle formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$.
3. Multiple-angle formulae.
4. Formulae for $\sin A \pm \sin B$, $\cos A \pm \cos B$.
5. Expression of $a \cos \theta + b \sin \theta$ in the forms $r \sin(\theta \pm \alpha)$ and $r \cos(\theta \pm \alpha)$, where r is positive, $0 < \alpha < \frac{\pi}{2}$.
6. General solution of simple trigonometric equations, including graphical interpretation.
7. Trigonometric identities $\cos^2 \theta + \sin^2 \theta \equiv 1$, $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$, $1 + \tan^2 \theta \equiv \sec^2 \theta$.
8. Maximum and minimum values of functions of $\sin \theta$ and $\cos \theta$.

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

SPECIFIC OBJECTIVES

(B) Co-ordinate Geometry

Students should be able to:

1. find equations of tangents and normals to circles;
2. find the points of intersection of a curve with a straight line;
3. find the points of intersection of two curves;
4. obtain the Cartesian equation of a curve given its parametric representation;
5. obtain the parametric representation of a curve given its Cartesian equation.

CONTENT

(B) Co-ordinate Geometry

1. Properties of the circle.
2. Tangents and normals.
3. Intersections between lines and curves.
4. Cartesian equations and parametric representations of curves including the parabola and ellipse.

SPECIFIC OBJECTIVES

(C) Vectors

Students should be able to:

1. express a vector in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $xi+yj+zk$
where i, j and k are unit vectors in directions of x -, y - and z -axis respectively;
2. define equality of two vectors;
3. add and subtract vectors;
4. multiply a vector by a scalar quantity;

UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

5. *derive and use unit vectors, position vectors and displacement vectors;*
6. *find the magnitude and direction of a vector;*
7. *find the angle between two given vectors using scalar product;*
8. *find the equation of a line in vector form, parametric form, Cartesian form, given a point on the line and a parallel vector to the line;*
9. *determine whether two lines are parallel, intersect, or skewed;*
10. *find the equation of the plane, in the form $xi + yj + zk = d$, $r \cdot n = d$, given a point in the plane and the normal to the plane.*

CONTENT

(C) Vectors

1. Expression of a given vector in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $xi + yj + zk$.
2. Equality, addition and subtraction of vectors; multiplication by a scalar.
3. Position vectors, unit vectors, displacement vectors.
4. Length (magnitude/modulus) and direction of a vector.
5. Scalar (Dot) Product.
6. *Vector equation of a line.*
7. *Equation of a plane.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. Trigonometric Identities

Much practice is required to master proofs of Trigonometric Identities using identities such as the formulae for:

$$\sin(A \pm B), \cos(A \pm B), \tan(A \pm B), \sin 2A, \cos 2A, \tan 2A$$



UNIT 1

MODULE 2: TRIGONOMETRY, GEOMETRY AND VECTORS (cont'd)

Example: The identity $\frac{1 - \cos 4\theta}{\sin 4\theta} \equiv \tan 2\theta$ can be established by realising that $\cos 4\theta \equiv 1 - 2 \sin^2 2\theta$ and $\sin 4\theta \equiv 2 \sin 2\theta \cos 2\theta$.

Derive the trigonometric functions $\sin x$ and $\cos x$ for angles x of any value (including negative values), using the coordinates of points on the unit circle.

2. Vectors

Teachers should introduce students to the three dimensional axis and understand how to plot vectors in three dimensions.

RESOURCE

- Bostock, L. and Chandler, S. *Mathematics - The Core Course for A-Level*, United Kingdom: Stanley Thornes (Publishers) Limited, 1997.
- Campbell, E. *Pure Mathematics for CAPE, Vol. 1, Jamaica*: LMH Publishing Limited, 2007.

UNIT 1

MODULE 3: CALCULUS I

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand the concept of continuity of a function from its graph;*
2. *develop the ability to find the limits (when they exist) of functions in simple cases;*
3. *know the relationships between the derivative of a function at a point and the behaviour of the function and its tangent at that point;*
4. *know the relationship between integration and differentiation;*
5. *know the relationship between integration and the area under the graph of the function;*
6. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) Limits

Students should be able to:

1. *use graphs to determine the continuity and discontinuity of functions;*
2. *describe the behaviour of a function $f(x)$ as x gets arbitrarily close to some given fixed number, using a descriptive approach;*
3. *use the limit notation $\lim_{x \rightarrow a} f(x) = L, f(x) \rightarrow L$ as $x \rightarrow a$;*
4. *use the simple limit theorems:*
If $\lim_{x \rightarrow a} f(x) = F, \lim_{x \rightarrow a} g(x) = G$ and k is a constant,
then $\lim_{x \rightarrow a} kf(x) = kF, \lim_{x \rightarrow a} f(x)g(x) = FG, \lim_{x \rightarrow a} \{f(x) + g(x)\} = F + G,$
and, provided $G \neq 0, \lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{F}{G}$;
5. *use limit theorems in simple problems, including cases in which the limit of $f(x)$ at a is not $f(a)$;*
6. *use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$, demonstrated by a geometric approach;*
7. *identify the point(s) for which a function is (un)defined;*
8. *identify the region over which a function is continuous;*

UNIT 1

MODULE 3: CALCULUS I (cont'd)

9. identify the point(s) where a function is discontinuous and describe the nature of its discontinuity;
10. use the concept of left-handed or right-handed *limit*, and continuity on a closed interval.

CONTENT

(A) Limits

1. Concept of limit of a function.
2. Limit Theorems.
3. Continuity and Discontinuity.

SPECIFIC OBJECTIVES

(B) Differentiation I

Students should be able to:

1. *define the derivative at a point as a limit;*
2. *differentiate, from first principles, functions such as:*
 - (a) $f(x) = k$ where $k \in \mathbb{R}$
 - (b) $f(x) = x^n$, where $n \in \{-3, -2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2, 3\}$,
 - (c) $f(x) = \sin x$,
 - (d) $f(x) = \cos x$.
3. use the product and quotient rules for differentiation;
4. differentiate products and quotients of:
 - (a) polynomials,
 - (b) trigonometric functions;
5. apply the chain rule in the differentiation of composite functions (substitution);

UNIT 1

MODULE 3: CALCULUS I (cont'd)

6. *solve problems involving rates of change;*
7. use the sign of the derivative to investigate where a function is increasing or decreasing;
8. *apply the concept of stationary (critical) points;*
9. determine the nature of stationary points;
10. *locate stationary points, maxima and minima including point(s) of inflexion by considering sign changes of the derivative;*
11. calculate second derivatives;
12. interpret the significance of the sign of the second derivative;
13. use the sign of the second derivative to determine the nature of stationary points;
14. *sketch graphs of polynomials, rational functions and trigonometric functions using the features of the function and its first and second derivatives (including horizontal and vertical asymptotes);*
15. describe the behaviour of such graphs for large values of the independent variable;
16. obtain equations of tangents and normals to curves.

CONTENT

(B) Differentiation I

1. The Gradient.
2. The Derivative as a limit.
3. Rates of change.
4. Differentiation from first principles.
5. Differentiation of simple functions, product, quotients.
6. Stationary points and chain rule.
7. Second derivatives of functions.
8. Curve sketching.
9. Tangents and Normals to curves.

UNIT 1
MODULE 3: CALCULUS I (cont'd)

SPECIFIC OBJECTIVES

(C) Integration I

Students should be able to:

1. *recognise integration as the reverse process of differentiation;*
2. demonstrate an understanding of the indefinite integral and the use of the integration notation $\int f(x) dx$;
3. show that the indefinite integral represents a family of functions which differ by constants;
4. demonstrate use of the following integration theorems:
 - (a) $\int cf(x) dx = c \int f(x) dx$, where c is a constant,
 - (b) $\int \{f(x) \pm g(x)\} dx = \int f(x) dx \pm \int g(x) dx$
5. find:
 - (a) indefinite integrals using integration theorems,
 - (b) integrals of polynomial functions,
 - (c) integrals of simple trigonometric functions;
6. integrate *using substitution*;
7. use the results:
 - (a) $\int_a^b f(x) dx = \int_a^b f(t) dt$,
 - (b) $\int_0^a f(x) dx = \int_0^a f(x - a) dx$ for $a > 0$,
 - (c) $\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$;

UNIT 1
MODULE 3: CALCULUS I (cont'd)

8. apply *integration* to:
 - (a) finding areas under the curve,
 - (b) finding areas between two curves,
 - (c) *finding volumes of revolution by rotating regions about both the x- and y-axes,*
 - (d) *finding volumes of revolution by rotating regions between two curves about both the x- and y-axes;*
9. *Given a rate of change with or without initial boundary conditions:*
 - (a) *formulate a differential equation of the form $y' = f(x)$ or $y'' = f(x)$ where f is a polynomial or a trigonometric function,*
 - (b) *solve the resulting differential equation and interpret the solution where applicable.*

CONTENT

(c) Integration I

1. Integration as the *reverse* of differentiation.
2. Linearity of integration.
3. Indefinite integrals (concept and use).
4. Definite integrals.
5. Applications of integration – areas, volumes and solutions to elementary differential equations.
6. Integration of polynomials.
7. Integration of simple trigonometric functions.
8. Use of $\int_a^b f(x) dx = F(b) - F(a)$, where $F'(x) = f(x)$.
9. *Simple first or second order differential equations of the type $y' = f(x)$ or $y'' = f(x)$.*

UNIT 1
MODULE 3: CALCULUS I (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

The Area under the Graph of a Continuous Function

Class discussion should play a major role in dealing with this topic. Activities such as that which follows may be performed to motivate the discussion.

Example of classroom activity:

Consider a triangle of area equal to $\frac{1}{2}$ units, bounded by the graphs of $y = x$, $y = 0$ and $x = 1$.

- (a) Sketch the graphs and identify the triangular region enclosed.
- (b) Subdivide the interval $[0, 1]$ into n equal subintervals.
- (c) Evaluate the sum, $s(n)$, of the areas of the inscribed rectangles and $S(n)$, of the circumscribed rectangles, erected on each subinterval.
- (d) By using different values of n , for example, for $n = 5, 10, 25, 50, 100$, show that both $s(n)$ and $S(n)$ get closer to the required area of the given region.

RESOURCES

- | | |
|--------------------------------|--|
| Bostock, L., and Chandler, S. | <i>Mathematics - The Core Course for A-Level, United Kingdom:</i> Stanley Thornes Publishing Limited, (Chapters 5, 8 and 9), 1991. |
| Campbell, E. | <i>Pure Mathematics for CAPE, Vol. 1, Jamaica:</i> LMH Publishing Limited, 2007. |
| Caribbean Examinations Council | <i>Area under the Graph of a Continuous Function,</i> Barbados: 1998. |
| Caribbean Examinations Council | <i>Differentiation from First Principles: The Power Function,</i> Barbados: 1998. |

◆ UNIT 2: COMPLEX NUMBERS, ANALYSIS AND MATRICES

MODULE 1: COMPLEX NUMBERS AND CALCULUS II

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop the ability to represent and deal with objects in the plane through the use of complex numbers;*
2. *be confident in using the techniques of differentiation and integration;*
3. *develop the ability to use concepts to model and solve real-world problems.*

SPECIFIC OBJECTIVES

(A) Complex Numbers

Students should be able to:

1. recognise the need to use complex numbers to find the roots of the general quadratic equation $ax^2 + bx + c = 0$, when $b^2 - 4ac < 0$;
2. *use the concept that complex roots occur in conjugate pairs;*
3. write the roots of the equation in that case and relate the sums and products to a , b and c ;
4. calculate the square root of a complex number;
5. express complex numbers in the form $a + bi$ where a , b are real numbers, and identify the real and imaginary parts;
6. add, subtract, multiply and divide complex numbers in the form $a + bi$, where a and b are real numbers;
7. find the principal value of the argument θ of a non-zero complex number, where $-\pi < \theta \leq \pi$;
8. find the modulus and conjugate of a given complex number;
9. interpret modulus and argument of complex numbers on the Argand Diagram;
10. represent complex numbers, their sums, differences and products on an Argand diagram;
11. find the set of all points z (locus of z) on the Argand Diagram such that z satisfies given properties;

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

12. apply De Moivre's theorem for integral values of n ;
13. establish and use $e^{ix} = \cos x + i \sin x$, for real x .

CONTENT

(A) Complex Numbers

1. Nature of roots of a quadratic equation, sums and products of roots.
2. *Conjugate pairs of complex roots.*
3. Addition, subtraction, multiplication and division of complex numbers in the form $a + bi$ where a, b are the real and imaginary parts, respectively, of the complex number.
4. The modulus, argument and conjugate of a complex number.
5. Representation of complex numbers on an Argand diagram.
6. Locus of a point.
7. De Moivre's theorem for integral n .
8. *Polar-argument and exponential forms of complex numbers.*

SPECIFIC OBJECTIVES

(B) Differentiation II

Students should be able to:

1. find the derivative of $e^{f(x)}$, where $f(x)$ is a differentiable function of x ;
2. find the derivative of $\ln f(x)$ (to include functions of x – polynomials or trigonometric);
3. apply the chain rule to obtain gradients of tangents and normals to curves given by their parametric equations;
4. use the concept of implicit differentiation, with the assumption that one of the variables is a function of the other;
5. differentiate any combinations of polynomials, trigonometric, exponential and logarithmic functions;
6. *find the first partial derivatives of $u = f(x, y)$ and $w = f(x, y, z)$;*

UNIT 2

MODULE 1: *COMPLEX NUMBERS AND CALCULUS II (cont'd)*

7. *find the second partial derivatives of $u = f(x, y)$ and $w = f(x, y, z)$;*
8. *differentiate inverse trigonometric functions;*
9. *obtain second derivatives, $f''(x)$, of the functions in 3, 4, 5 above.*

CONTENT

(B) Differentiation II

1. Application of the chain rule to differentiation.
2. Chain rule and differentiation of composite functions.
3. Gradients of tangents and normals.
4. Implicit differentiation.
5. First derivative of a function which is defined parametrically.
6. Differentiation of inverse trigonometric functions.
7. Differentiation of combinations of functions.
8. *First partial derivative.*
9. *Second partial derivative.*
10. Second derivative, that is, $f''(x)$.

SPECIFIC OBJECTIVES

(C) Integration II

Students should be able to:

1. express a rational function (*proper and improper*) in partial fractions in the cases where the denominator is of the form:

(a) $(ax + b)(cx + d)$, or

(b) $(ax + b)(cx + d)^2$, or

(c) $(ax + b)(cx^2 + d)$;

where a, b, c, d are integers.

UNIT 2

MODULE 1: *COMPLEX NUMBERS AND CALCULUS II (cont'd)*

2. express an improper rational function as a sum of a polynomial and partial fractions;
3. integrate rational functions in Specific Objectives 1 and 2 above;
4. integrate trigonometric functions using appropriate trigonometric identities;
5. integrate exponential functions and logarithmic functions;
6. find integrals of the form $\int \frac{f'(x)}{f(x)} dx$;
7. use substitutions to integrate functions (the substitution will be given in all but the most simple cases);
8. use integration by parts for combinations of functions;
9. integrate inverse trigonometric functions;
10. derive and use reduction formulae to obtain integrals;
11. use the trapezium rule as an approximation method for evaluating the area under the graph of the function.

CONTENT

(C) Integration II

1. Partial fractions.
2. Integration of rational functions, using partial fractions.
3. Integration by substitution.
4. Integration by parts.
5. Integration of inverse trigonometric functions.
6. Integration by reduction formula.
7. Area under the graph of a continuous function (Trapezium Rule).

UNIT 2

MODULE 1: COMPLEX NUMBERS AND CALCULUS II (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

Principal Argument of a Complex Number

The representation of the complex number $z = 1 + i$ on the Argand diagram may be used to introduce this topic. Encourage students to indicate and evaluate the argument of z . The students' answers should be displayed on the chalkboard.

Indicate that the location of z on the Argand diagram is unique, and therefore only one value of the argument is needed to position z . That argument is called the principal argument, $\arg z$, where:

$$-\pi < \text{principal argument} \leq \pi.$$

Students should be encouraged to calculate the principal argument by either solving:

(a) the simultaneous equations

$$\cos \theta = \frac{\operatorname{Re}(z)}{|z|} \text{ and } \sin \theta = \frac{\operatorname{Im}(z)}{|z|}, \text{ with } -\pi < \theta \leq \pi;$$

or,

(b) the equation

$$\tan \theta = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} \text{ for } \operatorname{Re}(z) \neq 0 \text{ and } -\pi < \theta \leq \pi,$$

together with the representation of z on the Argand diagram.

(c) Students should be encouraged to find the loci of z -satisfying equations such as:

(i) $|z - a| = k$;

(ii) $|z - c| = |z - b|$;

(iii) $\arg(z - a) = \alpha$.

RESOURCES

Bostock, L. and Chandler, S.

Core Mathematics for A-Levels, United Kingdom: Stanley Thornes Publishing Limited, 1997.

Bradie, B.

Rate of Change of Exponential Functions: A Precalculus Perspective, Mathematics Teacher Vol. 91(3), p. 224 – 237.

UNIT 2

MODULE 1: *COMPLEX NUMBERS AND CALCULUS II (cont'd)*

Campbell, E.

Pure Mathematics for CAPE, Vol. 2, Jamaica: LMH Publishing Limited, 2007.

Caribbean Examinations Council

The Exponential and Logarithmic Functions – An Investigation, Barbados: 1998.

Martin, A., Brown, K., Rigby, P. and Ridley, S.

Pure Mathematics, Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, 2000.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *understand* the concept of a sequence as a function from the natural numbers to the real numbers;
2. *understand* the difference between sequences and series;
3. distinguish between convergence and/or divergence of some standard series or sequences;
4. apply successive approximations to roots of equations;
5. *develop* the ability to use concept to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) Sequences

Students should be able to:

1. define the concept of a sequence $\{a_n\}$ of terms a_n as a function from the positive integers to the real numbers;
2. write a specific term from the formula for the n^{th} term, or from a recurrence relation;
3. describe the behaviour of convergent and divergent sequences, through simple examples;
4. apply mathematical induction to establish properties of sequences.

CONTENT

(A) Sequences

1. Definition, convergence, divergence, limit of a sequence.
2. Sequences defined by recurrence relations.
3. Application of mathematical induction to sequences.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

SPECIFIC OBJECTIVES

(B) Series

Students should be able to:

1. use the summation (Σ) notation;
2. define a series, as the sum of the terms of a sequence;
3. identify the n^{th} term of a series, in the summation notation;
4. define the m^{th} partial sum S_m as the sum of the first m terms of the sequence, that is,

$$S_m = \sum_{r=1}^m a_r;$$

5. apply mathematical induction to establish properties of series;
6. find the sum to infinity of a convergent series;
7. apply the method of differences to appropriate series, and find their sums;
8. use the Maclaurin theorem for the expansion of series;
9. *use the Taylor theorem for the expansion of series.*

CONTENT

(B) Series

1. Summation notation (Σ).
2. *Series as the sum of terms of a sequence.*
3. *Convergence and/or divergence of series to which the method of differences can be applied.*
4. *The Maclaurin series.*
5. *The Taylor series.*
6. Applications of mathematical induction to series.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

SPECIFIC OBJECTIVES

(C) The Binomial Theorem

Students should be able to:

1. explain the meaning and use simple properties of $n!$ and $\binom{n}{r}$, that is, ${}^n C_r$, where $n, r \in \mathbb{Z}$;
2. recognise that ${}^n C_r$, that is, $\binom{n}{r}$, is the number of ways in which r objects may be chosen from n distinct objects;
3. expand $(a + b)^n$ for $n \in \mathbb{Q}$;
4. apply the Binomial Theorem to real-world problems, for example, in mathematics of finance, science.

CONTENT

(C) The Binomial Theorem

1. Factorials and Binomial coefficients; their interpretation and properties.
2. The Binomial Theorem.
3. Applications of the Binomial Theorem.

SPECIFIC OBJECTIVES

(D) Roots of Equations

Students should be able to:

1. test for the existence of a root of $f(x) = 0$ where f is continuous using the Intermediate Value Theorem;
2. use interval bisection to find an approximation for a root in a given interval;
3. use linear interpolation to find an approximation for a root in a given interval;
4. explain, in geometrical terms, the working of the Newton-Raphson method;

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

5. use the Newton-Raphson method to find successive approximations to the roots of $f(x) = 0$, where f is differentiable;
6. use a given iteration to determine a root of an equation to a specified degree of accuracy.

CONTENT

(D) Roots of Equations

Finding successive approximations to roots of equations using:

1. *Intermediate Value Theorem;*
2. *Interval Bisection;*
3. *Linear Interpolation;*
4. *Newton - Raphson Method;*
5. *Iteration.*

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the learning activities listed below.

1. The Binomial Theorem

Students may be motivated to do this topic by having successive expansions of $(a + x)^n$ and then investigating the coefficients obtained when expansions are carried out.

$$(a + b)^1 = a + b$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + a^4$$

and so on.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

Similarly, the term in x^2 may be obtained by choosing two x 's and $(n-2)$ 1's. The x 's may be chosen from any two of the n brackets. This can be done in ${}^n C_2$ ways. The coefficient of x^2 is therefore ${}^n C_2$.

This process continues and the expansion is obtained:

$$(1+x)^n = 1 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + x^n.$$

This is known as the binomial theorem. The theorem may be written as

$$(1+x)^n = \sum_{r=0}^n {}^n C_r x^r.$$

The generalisation of this could be done as a class activity where students are asked to show that:

$$(a+b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + {}^n C_3 a^{n-3} b^3 + \dots + b^n.$$

This is the binomial expansion of $(a+b)^n$ for positive integral values of n . The expansion terminates after $(n+1)$ terms.

2. The Intermediate Value Theorem

(a) Motivate with an example.

Example: A taxi is travelling at 5 km/h at 8:00 a.m. Fifteen minutes later the speed is 100 km/h. Since the speed varies continuously, clearly at some time between 8:00 a.m. and 8:15 a.m. the taxi was travelling at 75 km/h.

Note that the taxi could have traveled at 75 km/h at more than one time between 8:00 a.m. and 8:15 a.m.

(b) Use examples of continuous functions to illustrate the Intermediate Value Theorem.

Example: $f(x) = x^2 - x - 6$ examined on the intervals $(3.5, 5)$ and $(0, 4)$.

3. Existence of Roots

Introduce the existence of the root of a continuous function $f(x)$ between given values a and b as an application of the Intermediate Value Theorem.

UNIT 2

MODULE 2: SEQUENCES, SERIES AND APPROXIMATIONS (cont'd)

Emphasis should be placed on the fact that:

- (a) f must be continuous between a and b ;
- (b) The product of $f(a)$ and $f(b)$ is less than zero, that is, $f(a)$ and $f(b)$ must have opposite signs.

4. *Interval Bisection*

Initially students should be able to determine an interval in which a real root lies. If $f(a)$ and $f(b)$ are of opposite signs, and f is continuous, then $a < x < b$, for the equation $f(x) = 0$.

Students may be asked to investigate $x = \frac{\alpha + \beta}{2}$ and note the resulting sign to determine which side of $\frac{\alpha + \beta}{2}$ the root lies. This method can be repeated until same answer to the desired degree of accuracy is obtained.

5. *Linear Interpolation*

*Given the points (x_0, y_0) and (x_1, y_1) on a continuous curve $y = f(x)$, students can establish that for $f(x_0)$ and $f(x_1)$ with opposite signs and that f is continuous, then $x_0 < x < x_1$, for the equation $f(x) = 0$. If $|f(x_0)| < |f(x_1)|$ say, students can be introduced to the concept of similar triangles to find successive approximations, holding $f(x_1)$ constant. This intuitive approach is formalised in **linear interpolation**, where the two points (x_0, y_0) and (x_1, y_1) can be joined by a straight line and the x -value of the point on this line is calculated. A first approximation for x can be found using*

$$\frac{x}{f(x_0)} = \frac{x_1}{f(x_1)}.$$

Successive approximations can be found with this approach until the same answer to the desired degree of accuracy is obtained.

RESOURCE

Bostock, L. and Chandler, S.

Core Mathematics for A-Levels, United Kingdom: Stanley Thornes Publishing Limited, 1997.

Campbell, E.

Pure Mathematics for CAPE, Vol. 2, Jamaica: LMH Publishing Limited, 2007.

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. *develop* the ability to analyse and solve simple problems dealing with choices and arrangements;
2. *develop an understanding of* the algebra of matrices;
3. *develop* the ability to analyse and solve systems of linear equations;
4. *develop skills to model some real-world phenomena by means of differential equations, and solve these;*
5. *develop* the ability to use concepts to model and solve real-world problems.

SPECIFIC OBJECTIVES

(A) Counting

Students should be able to:

1. state the principles of counting;
2. find the number of ways of arranging n distinct objects;
3. find the number of ways of arranging n objects some of which are identical;
4. find the number of ways of choosing r distinct objects from a set of n distinct objects;
5. identify a sample space;
6. *identify* the numbers of possible outcomes in a given sample space;
7. *use Venn diagrams to illustrate the principles of counting;*
8. *use possibility space diagram to identify a sample space;*
9. *define* and calculate $P(A)$, the probability of an event A occurring as the number of possible ways in which A can occur divided by the total number of possible ways in which all equally likely outcomes, including A , occur;
10. use the fact that $0 \leq P(A) \leq 1$;

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

11. demonstrate and use the property that the total probability for all possible outcomes in the sample space is 1;
12. use the property that $P(A') = 1 - P(A)$ is the probability that event A does not occur;
13. use the property $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for event A and B ;
14. use the property $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$, where A and B are mutually exclusive events;
15. use the property $P(A \cap B) = P(A) \times P(B)$, where A and B are independent events;
16. use the property $P(A|B) = \frac{P(A \cap B)}{P(B)}$ where $P(B) \neq 0$.
17. use a tree diagram to list all possible outcomes for conditional probability.

CONTENT

(A) Counting

1. Principles of counting.
2. Arrangements with and without repetitions.
3. Selections.
4. Venn diagram.
5. Possibility space diagram.
6. Concept of probability and elementary applications.
7. Tree diagram.

SPECIFIC OBJECTIVES

(B) Matrices and Systems of Linear Equations

Students should be able to:

1. operate with conformable matrices, carry out simple operations and manipulate matrices using their properties;
2. evaluate the determinants of $n \times n$ matrices, $1 \leq n \leq 3$;

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

3. reduce a system of linear equations to echelon form;
4. row-reduce the augmented matrix of an $n \times n$ system of linear equations, $n = 2, 3$;
5. determine whether the system is consistent, and if so, how many solutions it has;
6. find all solutions of a consistent system;
7. invert a non-singular 3×3 matrix;
8. solve a 3×3 system of linear equations, having a non-singular coefficient matrix, by using its inverse.

CONTENT

(B) Matrices and Systems of Linear Equations

1. $m \times n$ matrices, for $1 \leq m \leq 3$, and $1 \leq n \leq 3$, and equality of matrices.
2. Addition of conformable matrices, zero matrix and additive inverse, associativity, commutativity, distributivity, transposes.
3. Multiplication of a matrix by a scalar.
4. Multiplication of conformable matrices.
5. Square matrices, singular and non-singular matrices, unit matrix and multiplicative inverse.
6. $n \times n$ determinants, $1 \leq n \leq 3$.
7. $n \times n$ systems of linear equations, consistency of the systems, equivalence of the systems, solution by reduction to row echelon form, $n = 2, 3$.
8. $n \times n$ systems of linear equations by row reduction of an augmented matrix, $n = 2, 3$.

SPECIFIC OBJECTIVES

(C) *Differential Equations and Modeling*

Students should be able to:

1. solve first order linear differential equations $y' - ky = f(x)$ using an integrating factor, given that k is a real constant or a function of x , and f is a function;

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

2. solve first order linear differential equations given boundary conditions;
3. solve second order ordinary differential equations with constant coefficients of the form

$$ay'' + by' + cy = 0 = f(x), \text{ where } a, b, c \in \mathbb{R} \text{ and } f(x) \text{ is:}$$

- (a) a polynomial,
- (b) an exponential function,
- (c) a trigonometric function;

and the complementary function may consist of

- (a) 2 real and distinct roots;
 - (b) 2 equal roots;
 - (c) 2 complex roots.
4. solve second order ordinary differential equation given boundary conditions;
 5. use substitution to reduce a second order ordinary differential equation to a suitable form.

CONTENT

(C) Differential Equations and Modeling

1. Formulation and solution of differential equations of the form $y' - ky = f(x)$, where k is a real constant or a function of x , and f is a function.
2. Second order ordinary differential equations.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives of this Module, teachers are advised to engage students in the teaching and learning activities listed below.

1. Counting

Consider the three scenarios given below.

- (a) Throw two dice. Find the probability that the sum of the dots on the uppermost faces of the dice is 6.
- (b) An insurance salesman visits a household. What is the probability that he will be successful in selling a policy?

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

- (c) A hurricane is situated 500km east of Barbados. What is the probability that it will hit the island?

These three scenarios are very different for the calculation of probability. In 'a', the probability is calculated as the number of successful outcomes divided by the total possible number of outcomes. In this classical approach, the probability assignments are based on equally likely outcomes and the entire sample space is known from the start.

The situation in 'b' is no longer as well determined as in 'a'. It is necessary to obtain historical data for the salesman in question and estimate the required probability by dividing the number of successful sales by the total number of households visited. This frequency approach still relies on the existence of data and its applications are more realistic than those of the classical methodology.

For 'c' it is very unclear that a probability can be assigned. Historical data is most likely unavailable or insufficient for the frequency approach. The statistician might have to revert to informed educated guesses. This is quite permissible and reflects the analyst's prior opinion. This approach lends itself to a Bayesian methodology.

One should note that the rules and results of probability theory remain exactly the same regardless of the method used to estimate the probability of events.

2. Systems of Linear Equations in Two Unknowns

- (a) In order to give a geometric interpretation, students should be asked to plot on graph paper the pair of straight lines represented by a given pair of linear equations in two unknowns, and to examine the relationship between the pair of straight lines in the cases where the system of equations has been shown to have:
- (i) one solution;
 - (ii) many solutions;
 - (iii) no solutions.
- (b) Given a system of equations with a unique solution, there exist equivalent systems, obtained by row-reduction, having the same solution. To demonstrate this, students should be asked to plot on the same piece of graph paper all the straight lines represented by the successive pairs of linear equations which result from each of the row operations used to obtain the solution.

UNIT 2

MODULE 3: COUNTING, MATRICES AND DIFFERENTIAL EQUATIONS (cont'd)

RESOURCES

- Bolt, B. and Hobbs, D. *101 Mathematical Projects: A Resource Book*, United Kingdom: Cambridge University Press, 1994.
- Bostock, L. and Chandler, S. *Core Mathematics for A-Levels*, United Kingdom: Stanley Thornes Publishing Limited, 1997.
- Campbell, E. *Pure Mathematics for CAPE, Vol. 2, Jamaica*: LMH Publishing Limited, 2007.
- Crawshaw, J. and Chambers, J. *A Concise Course in A-Level Statistics*, Cheltenham, United Kingdom: Stanley Thornes (Publishers) Limited, 1999.

◆ OUTLINE OF ASSESSMENT

Each Unit of the syllabus is assessed separately. The scheme of assessment for each Unit is the same. A candidate's performance on each Unit is reported as an overall grade and a grade on each Module of the Unit. The assessment comprises two components, one external and one internal.

EXTERNAL ASSESSMENT (80%)

The candidate is required to sit two written papers for a total of 4 hrs.

Paper 01 (1 hour 30 minutes)	This paper comprises forty-five, compulsory multiple-choice items.	30%
Paper 02 (2 hours 30 minutes)	This paper comprises six, compulsory extended-response questions.	50%

SCHOOL-BASED ASSESSMENT (20%)

School-Based Assessment in respect of each Unit will contribute 20 per cent to the total assessment of a candidate's performance on that Unit.

Paper 03/1

This paper is intended for candidates registered through a school or other approved educational institution.

The School-Based Assessment comprises three class tests designed and assessed internally by the teacher and externally by CXC. The duration of each test is 1 to 1½ hours. The tests must span, individually or collectively, the three Modules, and must include mathematical modelling.

Paper 03/2 (Alternative to Paper 03/1)

This paper is an alternative to Paper 03/1 and is intended for private candidates.

The paper comprises three questions. The duration of the paper is 1½ hours.

MODERATION OF SCHOOL-BASED ASSESSMENT (PAPER 03/1)

Each year a School-Based Assessment Record Sheet will be sent to each school submitting candidates for the examinations.

All School-Based Assessment Record Sheets and samples of tests from the school must be submitted to CXC by 31 May of the year of the examination. A sample of tests must be submitted to CXC for moderation purposes. The tests will be re-assessed by CXC Examiners who moderate the School-Based Assessment. The teachers' marks may be adjusted as a result of the moderation. The Examiners' comments will be sent to the teacher.

Copies of the candidates' assignments must be retained by the school until three months after publication by CXC of the examination results.

ASSESSMENT DETAILS FOR EACH UNIT

External Assessment by Written Papers (80 per cent of Total Assessment)

Paper 01 (1 hour 30 minutes – 30 per cent of Total Assessment)

1. Composition of the Paper

- (a) This paper consists of forty-five multiple-choice items, with fifteen items based on each Module.
- (b) All items are compulsory.

2. Syllabus Coverage

- (a) Knowledge of the entire syllabus is required.
- (b) The paper is designed to test a candidate's knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

- (a) Each item is allocated 1 mark.
- (b) Each Module is allocated 15 marks.
- (c) The total marks available for this paper is 45.
- (d) This paper contributes 30 per cent towards the final assessment.

5. Award of Marks

Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

Reasoning: Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

Algorithmic Knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.

Conceptual Knowledge: Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.

6. Use of Calculators

- (a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is entirely responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.
- (d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae will be provided.

Paper 02 (2 hours 30 minutes – 50 per cent of Total Assessment)

1. Composition of Paper

- (a) The paper consists of six questions. Two questions are based on each Module (Module 1, Module 2 and Module 3).
- (b) All questions are compulsory.

2. Syllabus Coverage

- (a) Each question may be based on one or more than one topic in the Module from which the question is taken.
- (b) Each question may develop a single theme or unconnected themes.

3. Question Type

- (a) Questions may require an extended response.
- (b) Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

- (a) Each question is worth 25 marks.
- (b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.
- (c) Each Module is allocated 50 marks.
- (d) The total marks available for this paper is 150.
- (e) This paper contributes 50 per cent towards the final assessment.

5. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

Reasoning: Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.

Algorithmic Knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.

Conceptual Knowledge: Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.

- (b) Full marks will be awarded for correct answers and presence of appropriate working.
- (c) Where an incorrect answer is given, credit may be awarded for correct method provided that the working is shown.
- (d) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the latter part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (e) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates are, therefore, advised to show all relevant working.

6. Use of Calculators

- (a) Each candidate is required to have a silent, non-programmable calculator for the duration of the examination, and is responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.
- (d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae will be provided.

SCHOOL-BASED ASSESSMENT

School-Based Assessment is an integral part of student assessment in the course covered by this syllabus. It is intended to assist students in acquiring certain knowledge, skills, and attitudes that are associated with the subject. The activities for the School-Based Assessment are linked to the syllabus and should form part of the learning activities to enable the student to achieve the objectives of the syllabus.

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. In order to ensure that the scores awarded by teachers are in line with the CXC standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of students. It facilitates feedback to the students at various stages of their experience. This helps to build the self-confidence of students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities emphasised by this CAPE subject and enhance the validity of the examination on which candidate performance is reported. School-Based assessment, therefore, makes a significant and unique contribution to both the development of relevant skills and the testing and rewarding of students for the development of those skills.

The Caribbean Examinations Council seeks to ensure that the School-Based Assessment scores are valid and reliable estimates of accomplishment. The guidelines provided in this syllabus are intended to assist in doing so.

Paper 03/1 (20 per cent of Total Assessment)

This paper comprises three tests. The tests, designed and assessed by the teacher, are externally moderated by CXC. The duration of each test is 1 to 1½ hours.

1. Composition of the Tests

The three tests of which the School-Based Assessment is comprised must span, individually or collectively, the three Modules and include mathematical modelling. At least 30 per cent of the marks must be allocated to mathematical modelling.

2. Question Type

Paper 03/2 may be used as a prototype but teachers are encouraged to be creative and original.

3. Mark Allocation

- (a) There is a maximum of 20 marks for each test.
- (b) There is a maximum of 60 marks for the School-Based Assessment.

- (c) The candidate's mark is the total mark for the three tests. One-third of the total marks for the three tests is allocated to each of the three Modules. (See 'General Guidelines for Teachers' below.)
- (d) For each test, marks should be allocated for the skills outlined on page 3 of this Syllabus.

4. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

For each test, the 20 marks should be awarded as follows:

Reasoning:	Selection of appropriate strategy, evidence of clear thinking, explanation and/or logical argument.	(3 – 5 marks)
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Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.	(10 – 14 marks)
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Conceptual Knowledge:	Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams.	(3 – 5 marks)
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- (b) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (c) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates should be advised to show all relevant working.

Paper 03/2 (20 per cent of Total Assessment)

1. Composition of Paper

- (a) This paper consists of three questions, each based on one of the three Modules.
- (b) All questions are compulsory.

2. Question Type

- (a) Each question may require an extended response.
- (b) A part of or an entire question may focus on mathematical modeling.
- (c) A question may be presented using words, symbols, tables, diagrams or a combination of these.

3. Mark Allocation

- (a) Each question carries a maximum of 20 marks.
- (b) The Paper carries a maximum of 60 marks.
- (c) For each question, marks should be allocated for the skills outlined on page 3 of this Syllabus.

4. Award of Marks

- (a) Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

For each test, the 20 marks should be awarded as follows:

Reasoning:	Selection of appropriate strategy, evidence of clear reasoning, explanation and/or logical argument. (3 – 5 marks)
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Algorithmic Knowledge:	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner. (10 – 14 marks)
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Conceptual Knowledge:	Recall or selection of facts or principles; computational skill, numerical accuracy, and acceptable tolerance limits in drawing diagrams. (3 – 5 marks)
-----------------------	--

- (b) If an incorrect answer in a previous question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (c) A correct answer given with no indication of the method used (in the form of written working) will receive no marks. Candidates should be advised to show all relevant working.

GENERAL GUIDELINES FOR TEACHERS

1. Marks must be submitted to CXC on a yearly basis on the School-Based Assessment forms provided. The forms should be despatched through the Local Registrar for submission to CXC by 31 May in Year 1 and 31 May in Year 2.
2. The School-Based Assessment for each year should be completed in duplicate. The original should be submitted to CXC and the copy retained by the school.
3. CXC will require a sample of the tests for external moderation. These tests must be retained by the school for at least three months after publication of examination results.

4. Teachers should note that the reliability of marks awarded is a significant factor in the *School-Based Assessment*, and has far-reaching implications for the candidate's final grade.
5. Candidates who do not fulfill the requirements of the *School-Based Assessment* will be considered absent from the whole examination.
6. Teachers are asked to note the following:
 - (a) the relationship between the marks for the assignment and those submitted to CXC on the *school-based assessment* form should be clearly shown;
 - (b) the teacher is required to allocate one-third of the total score for the *School-Based Assessment* to each Module. Fractional marks should not be awarded. In cases where the mark is not divisible by three, then:
 - (i) when the remainder is 1 mark, the mark should be allocated to Module 3;
 - (ii) when the remainder is 2, then a mark should be allocated to Module 3 and the other mark to Module 2;for example, 35 marks would be allocated as follows:
 $35/3 = 11$ remainder 2 so 11 marks to Module 1 and 12 marks to each of Modules 2 and 3.
 - (c) the standard of marking should be consistent.
7. Teachers are required to submit a copy of EACH test, the solutions and the mark schemes with the sample.

◆ REGULATIONS FOR PRIVATE CANDIDATES

Candidates who are registered privately will be required to sit Paper 01, Paper 02 and Paper 03/2. Paper 03/2 will be 1½ hours' duration and will consist of three questions, each worth 20 marks. Each question will be based on the objectives and content of one of the three Modules of the Unit. Paper 03/2 will contribute 20 per cent of the total assessment of a candidate's performance on that Unit.

Paper 03/2 (1 hour 30 minutes)

The paper consists of three questions. Each question is based on the topics contained in one Module and tests candidates' skills and abilities to:

1. recall, select and use appropriate facts, concepts and principles in a variety of contexts;
2. manipulate mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences;
3. select and use a simple mathematical model to describe a real-world situation;
4. simplify and solve mathematical models;
5. interpret mathematical results and their application in a real-world problem.

◆ REGULATIONS FOR RE-SIT CANDIDATES

Candidates, who have earned a moderated score of at least 50 per cent of the total marks for the Internal Assessment component, may elect not to repeat this component, provided they re-write the examination no later than TWO years following their first attempt. These resit candidates must complete Papers 01 and 02 of the examination for the year in which they register.

Resit candidates must be entered through a school or other approved educational institution.

Candidates who have obtained less than 50 per cent of the marks for the School-Based Assessment component must repeat the component at any subsequent sitting or write Paper 03/2.

◆ ASSESSMENT GRID

The Assessment Grid for each Unit contains marks assigned to papers and to Modules and percentage contributions of each paper to total scores.

Units 1 and 2

Papers	Module 1	Module 2	Module 3	Total	(%)
External Assessment Paper 01 (1 hour 30 minutes)	15 (30 weighted)	15 (30 weighted)	15 (30 weighted)	45 (90 weighted)	(30)
Paper 02 (2 hours 30 minutes)	50	50	50	150	(50)
School-Based Assessment Paper 03/1 or Paper 03/2 (1 hour 30 minutes)	20	20	20	60	(20)
Total	100	100	100	300	(100)

◆ MATHEMATICAL NOTATION

The following list summarises the notation used in the Mathematics papers of the Caribbean Advanced Proficiency Examination.

Set Notation

\in	is an element of
\notin	is not an element of
$\{x: \dots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
\emptyset	the empty set
U	the universal set
A'	the complement of the set A
\mathbb{W}	the set of whole numbers $\{0, 1, 2, 3, \dots\}$
\mathbb{N}	the set of natural numbers $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers
\mathbb{Q}	the set of rational numbers
$\overline{\mathbb{Q}}$	the set of irrational numbers
\mathbb{R}	the set of real numbers
\mathbb{C}	the set of complex numbers
\subset	is a <i>proper</i> subset of
$\not\subset$	is not a proper subset of
\subseteq	is a subset of
$\not\subseteq$	is not a subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$

Logic

\wedge	conjunction
\vee	(inclusive) injunction
\veebar	exclusive disjunction
\sim	negation
\rightarrow	conditionality
\leftrightarrow	bi-conditionality
\Rightarrow	implication
\Leftrightarrow	equivalence

Miscellaneous Symbols

\equiv	is identical to
\approx	is approximately equal to
\propto	is proportional to
∞	infinity

Operations

$$\sum_{i=1}^n x_i$$

$$x_1 + x_2 + \dots + x_n$$

$$\sqrt{x}$$

the positive square root of the real number x

$$|x|$$

the modulus of the real number x

$$n!$$

n factorial, $1 \times 2 \times \dots \times n$ for $n \in \mathbb{N}$ ($0! = 1$)

$${}^n C_r \binom{n}{r}$$

the binomial coefficient, $\frac{n!}{(n-r)!r!}$, for $n, r \in \mathbb{N}$, $0 \leq r \leq n$

$${}^n P_r$$

$$\frac{n!}{(n-r)!}$$

Functions

f

the function f

$f(x)$

the value of the function f at x

$f: A \rightarrow B$

the function f under which each element of the set A has an image in the set B

$f: x \rightarrow y$

the function f maps the element x to the element y

f^{-1}

the inverse of the function f

fg

the composite function $f(g(x))$

$\lim_{x \rightarrow a} f(x)$

the limit of $f(x)$ as x tends to a

$\Delta x, \delta x$

an increment of x

$\frac{dy}{dx}, y'$

the *first* derivative of y with respect to x

$\frac{d^n y}{dx^n}, y^{(n)}$

the n th derivative of y with respect to x

$f'(x), f''(x), \dots, f^{(n)}(x)$

the first, second, \dots , n th derivatives of $f(x)$ with respect to x

\dot{x}, \ddot{x}

the first and second derivatives of x with respect to time t

e

the exponential constant

$\ln x$

the natural logarithm of x (to base e)

$\lg x$

the logarithm of x to base 10

Complex Numbers

i

$$\sqrt{-1}$$

z

a complex number, $z = x + yi$ where $x, y \in \mathbb{P}$

$\operatorname{Re} z$

the real part of z

$\operatorname{Im} z$

the imaginary part of z

$|z|$

the modulus of z

$\arg z$

the argument of z , where $-\pi < \arg z \leq \pi$

\bar{z}, z^*

the complex conjugate of z

Vectors

\vec{a} , \mathbf{a} , \overrightarrow{AB}

$\hat{\mathbf{a}}$
 $|\mathbf{a}|$

$\mathbf{a} \cdot \mathbf{b}$

\mathbf{i} , \mathbf{j} , \mathbf{k}

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

vectors

a unit vector in the direction of the vector \mathbf{a}

the magnitude of the vector \mathbf{a}

the scalar product of the vectors \mathbf{a} and \mathbf{b}

unit vectors in the directions of the positive Cartesian coordinate axes

$x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

Probability

S

A , B , ...

$P(A')$

the sample space

the events A , B , ...

the probability that the event A does not occur

Matrices

M

(M^{-1})

M^T , M_T

$\det M$, $|M|$

a matrix M

inverse of the non-singular square matrix M

transpose of the matrix M

determinant of the square matrix M